Using Ambient Vibration Array Techniques for Site Characterisation

Dispersion Curve Inversion

Lecture

Dispersion Curve Inversion

- What's an inverse problem?
- Inversion techniques
- Neighbourhood Algorithm (NA, Sambridge, 1999)
- Conditional parameter spaces
- Dispersion curve inversion examples

SESARRAY PACKAGE

GEOPSY

array tools

H/V

damping

FK

Capon

MSPAC

MASW

figue

figures

gp tools

Dispersion curves

Ellipticity curves

Autocorr. Curves

…

build_array

Array response

Post-processing

max2curve

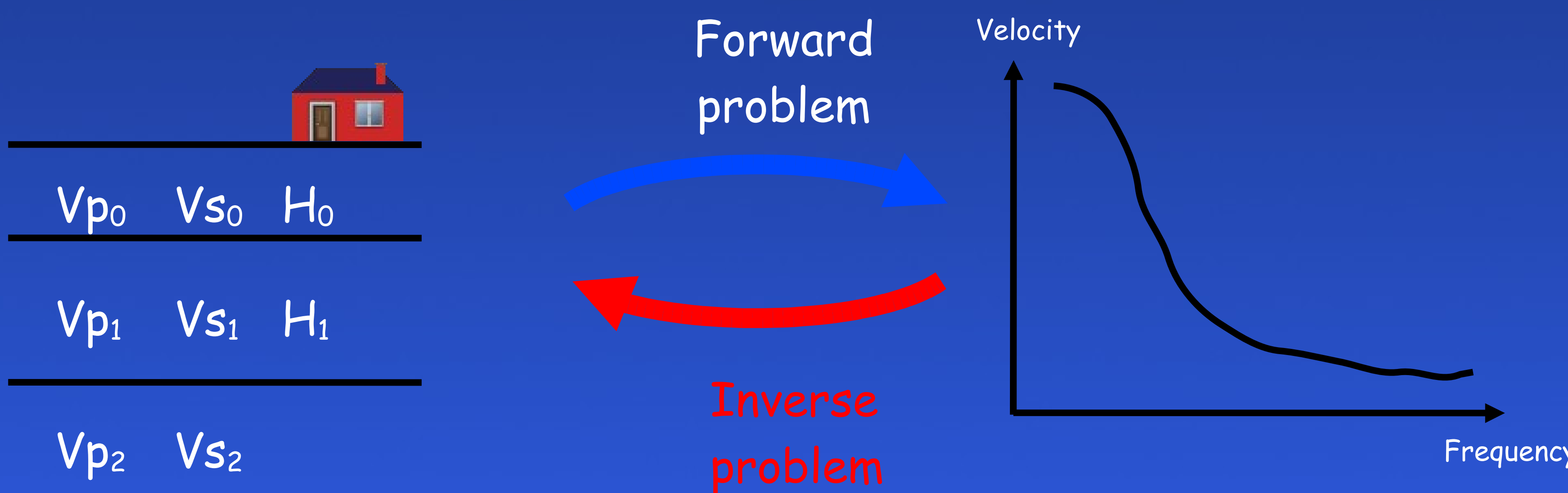
spac2disp

DINVER

inversion

1. What's an inversion problem ?

Parameters



Ground model

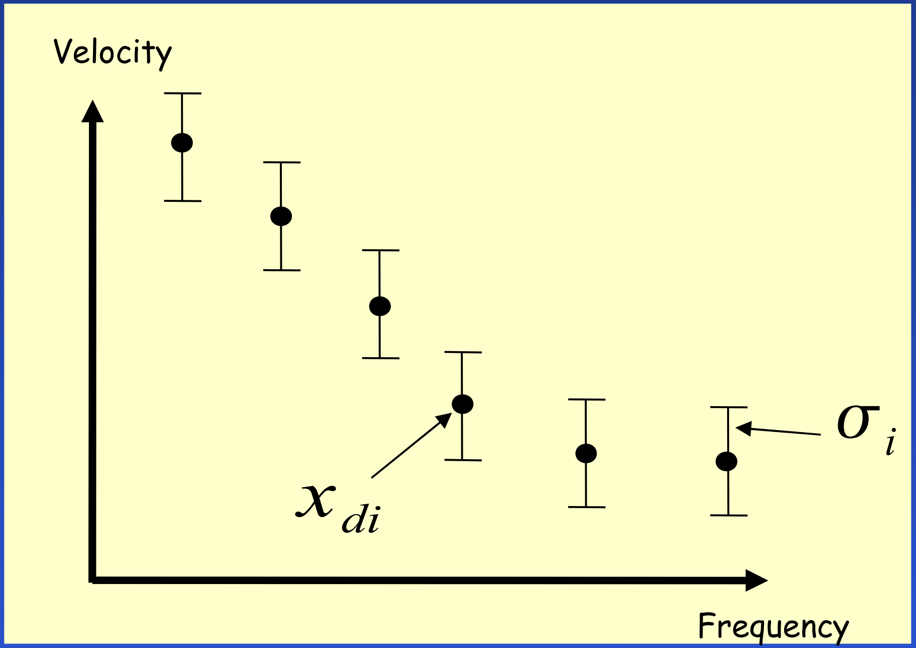
Dispersion curve or SPAC ratios

Unknowns

Data

2. Inversion Techniques

Ranking models
vs
Inversion target

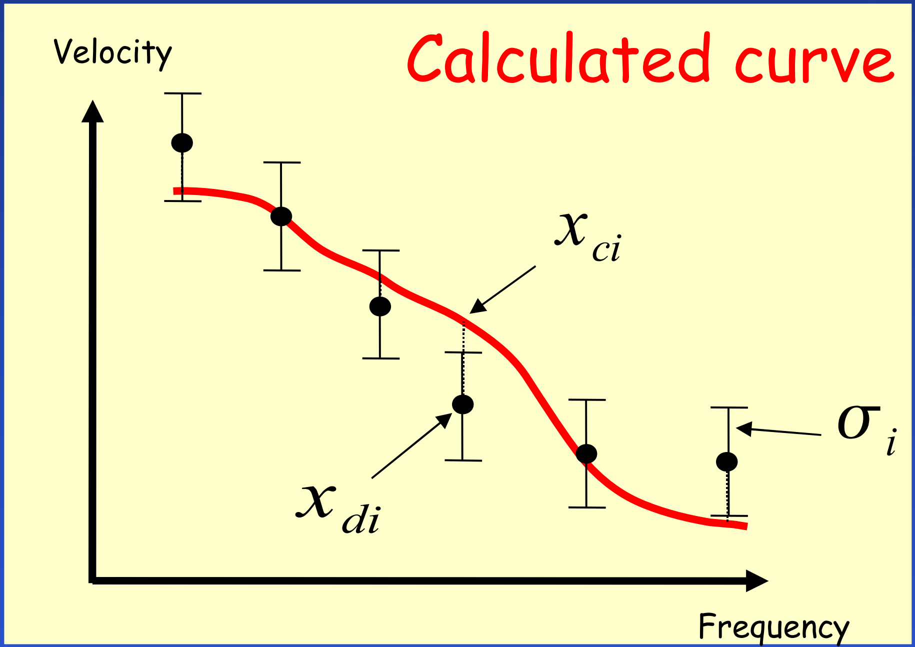


$$\text{Misfit} = \sqrt{\sum_{i=1}^{n_F} \frac{(x_{di} - x_{ci})^2}{\sigma_i^2 n_F}}$$

n_F Number of frequency samples

2. Inversion Techniques

Ranking models
vs
Inversion target



$$\text{Misfit} = \sqrt{\sum_{i=1}^{n_F} \frac{(x_{di} - x_{ci})^2}{\sigma_i^2 n_F}}$$

n_F Number of frequency samples

A gentle 2D misfit function…

Imagine yourself without a map (nor a GPS) …

... at the same place on a stormy day.

Where is the exit ? (= minimum misfit)

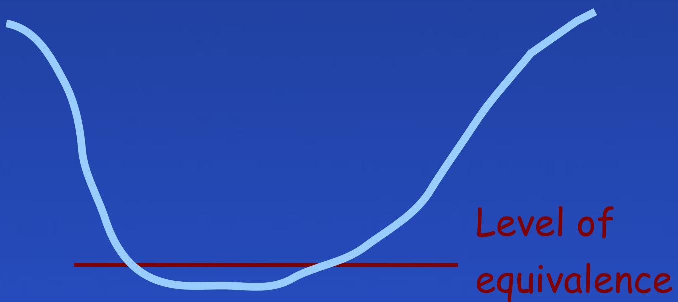
Where is the exit ?

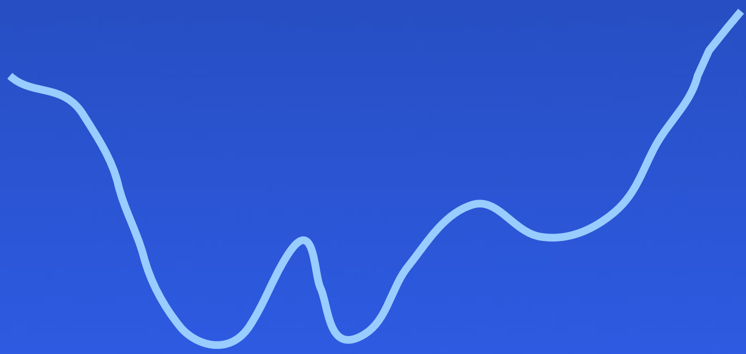
Start from anywhere and go down?

Possible shapes for a misfit function

A single narrow valley

A larger valley





Local minima and
equivalent minima

A truly flat valley

2. Inversion Techniques

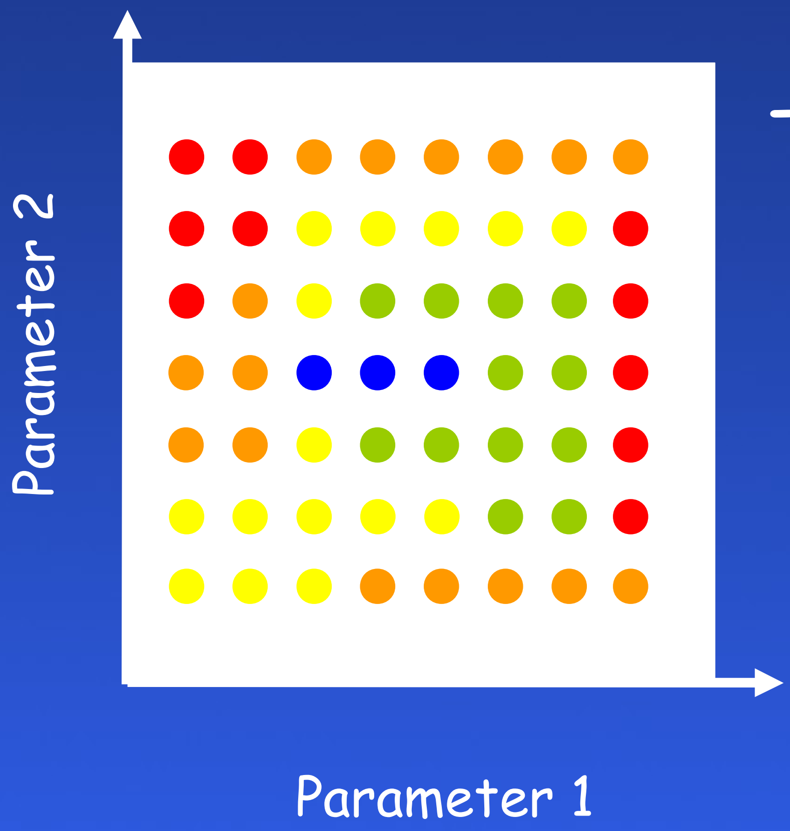
Forward problem:

- Analytic or numerical processing
- Only one solution

Inverse problem:

- Trial and error to adjust parameters of the model
- Simplex downhill method
- Brute force uniform search (gridding)
- Least square methods (based on derivatives)
- Brute force Monte Carlo sampling
- Simulated Annealing
- Genetic Algorithm
- Neighbourhood Algorithm
- Generally not only one solution

A. Uniform search (gridding)

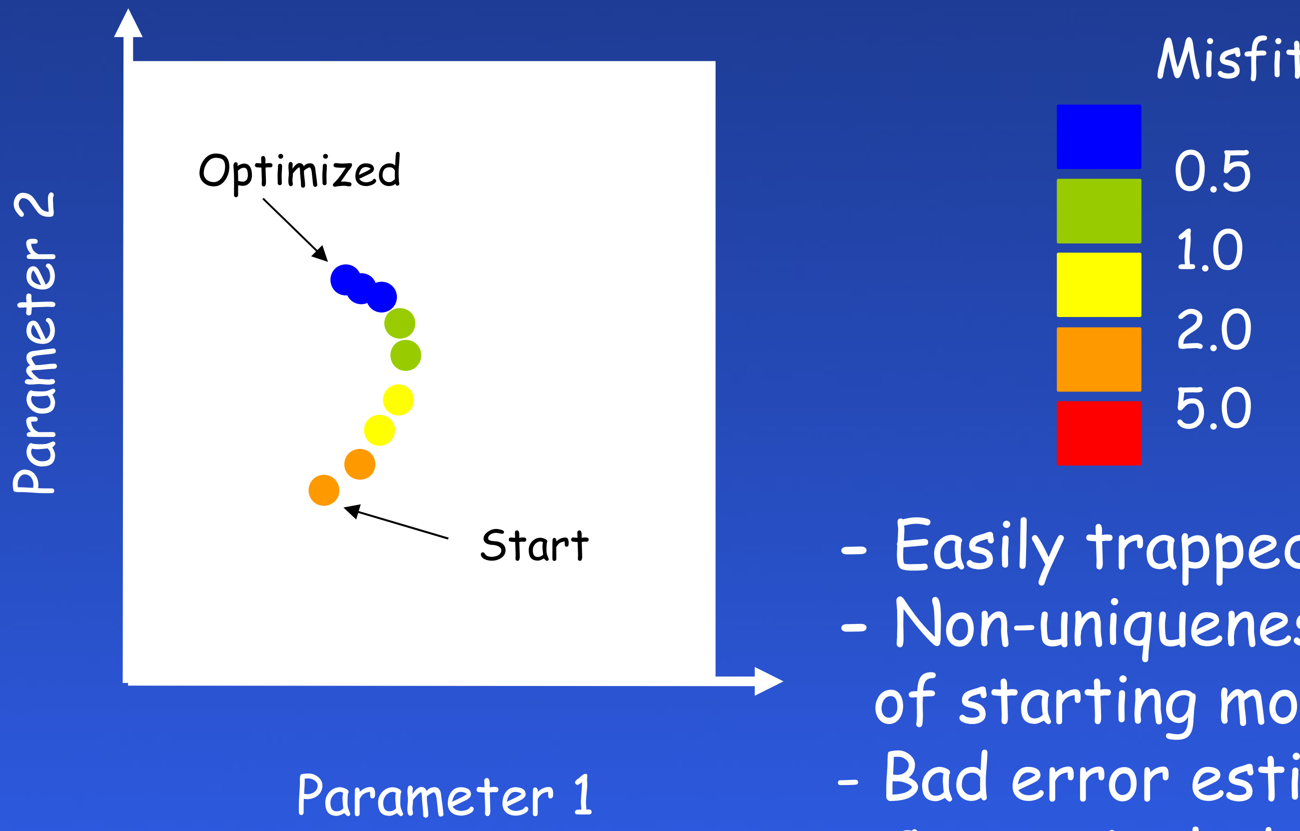


- If nd > 3 : number of forward computations are prohibitive

+ Complete exploration of the parameter space
+ Optimum error estimates

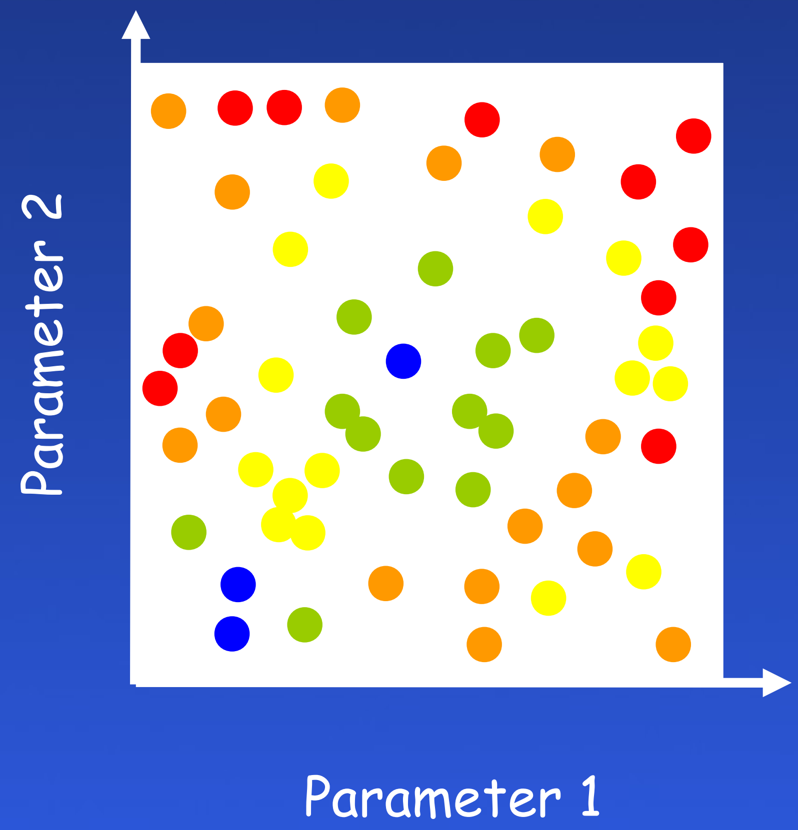
B. Iterative methods (optimisation)

Least Square, Simplex, Gradient methods, ...



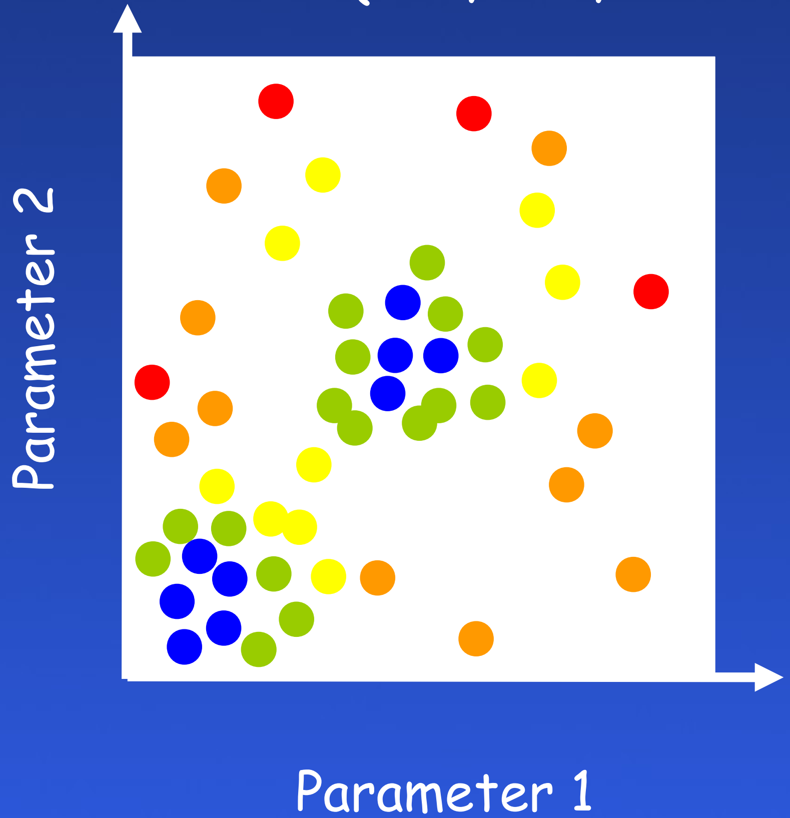
- Easily trapped in local minima
- Non-uniqueness <=> choice of starting model
- Bad error estimates
- Cannot include prior information
+ High dimensionality
+ Few forward computations

C. Random search (Monte Carlo)



- Requires lot of forward computations
+ Not too bad exploration of the parameter space
+ Good error estimates

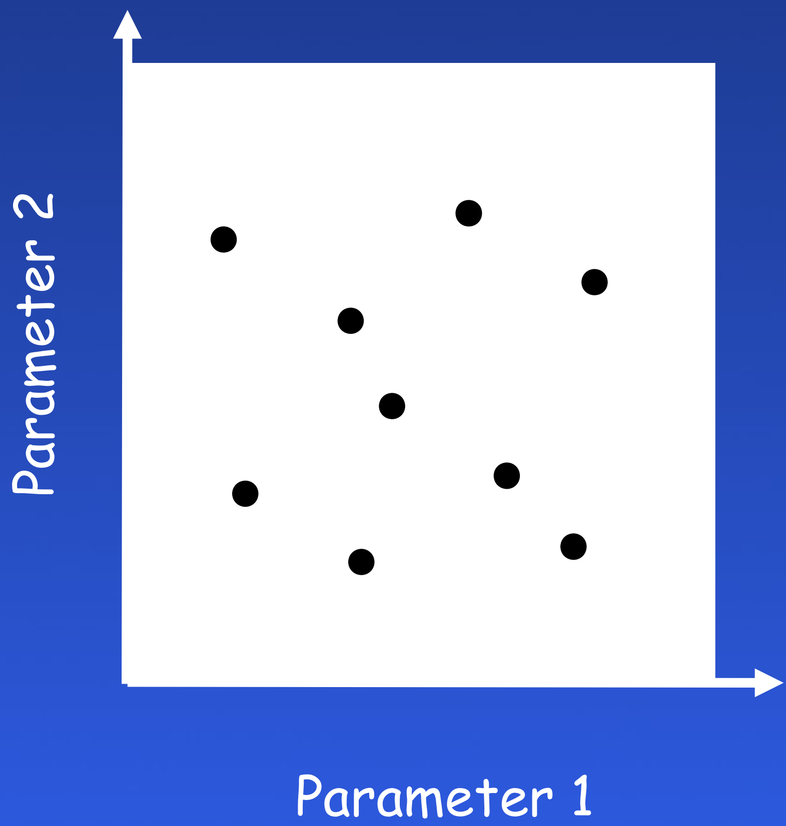
D. Oriented random search (~ 1990) (SA, GA, and NA)



+ Requires less of forward computations than MC
- Max nd ~ 25-50
+ Not too bad exploration of the parameter space
+ Good error estimates

3. Neighbourhood Algorithm

(Sambridge, 1999)



Few tuning parameters (Ns, Nr)

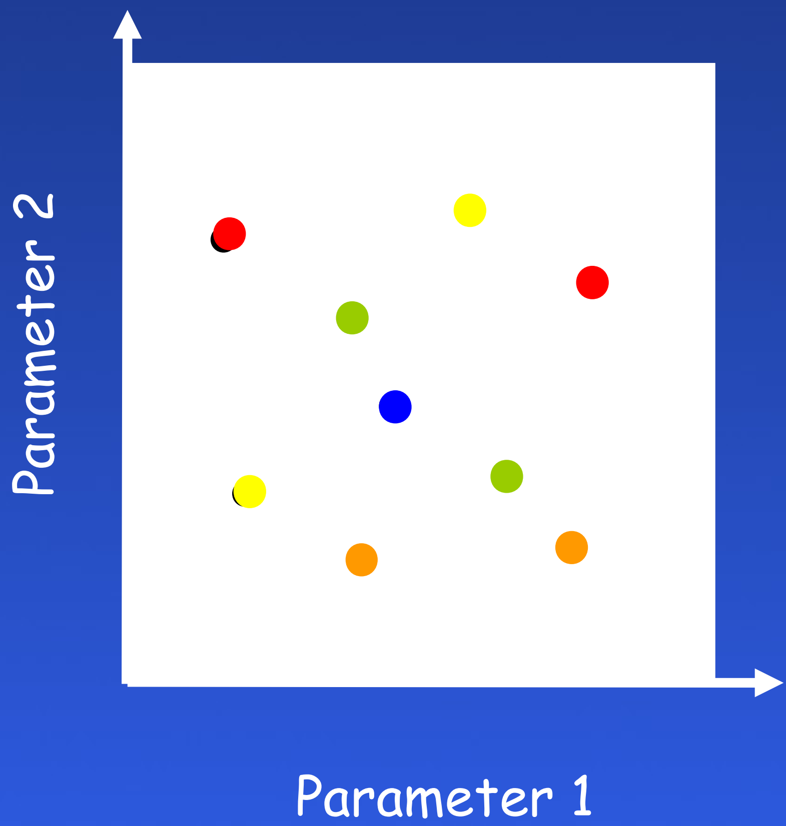
Based on Voronoi division of the parameter space

~ SA and GA (better according to Sambridge)

Ns new samples generated into Nr selected cells

3. Neighbourhood Algorithm

(Sambridge, 1999)



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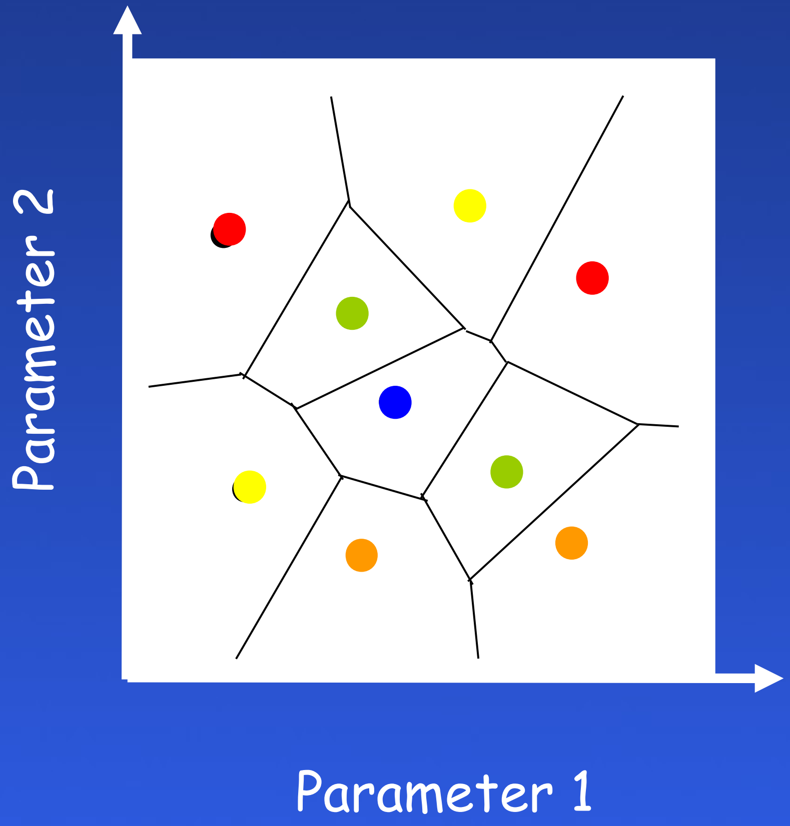

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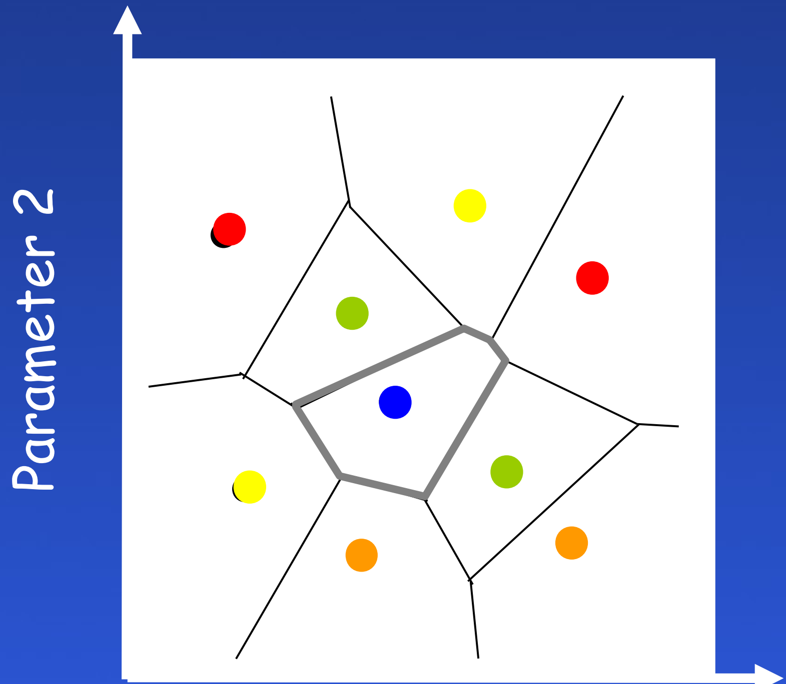

Few tuning parameters (Ns, Nr)

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Misfit

0.5

1.0

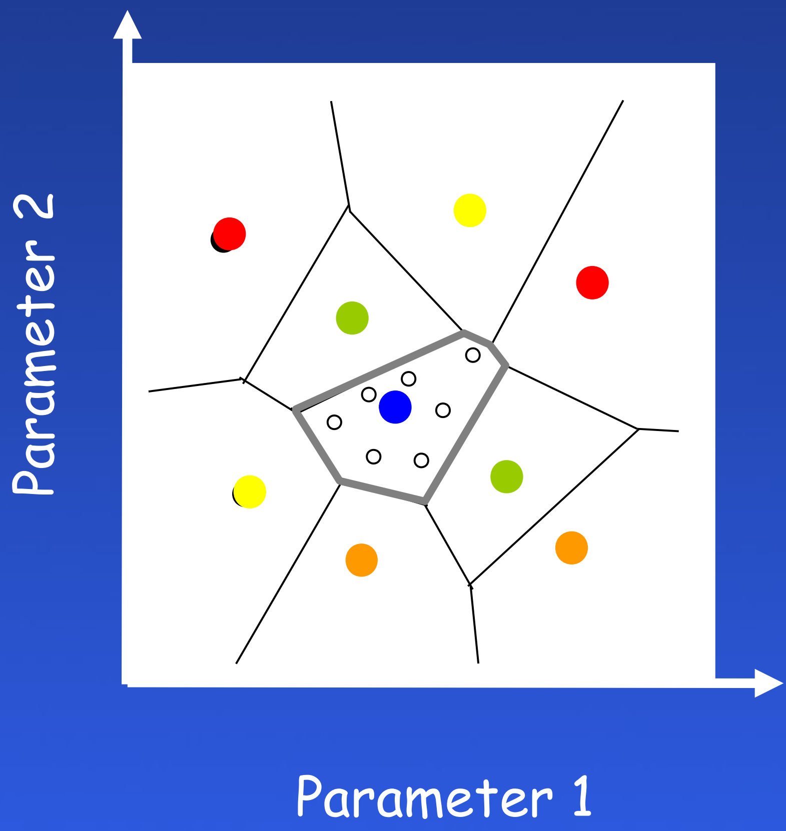
2.0

5.0

Ns new samples generated into
Nr selected cells

3. Neighbourhood Algorithm

(Sambridge, 1999)



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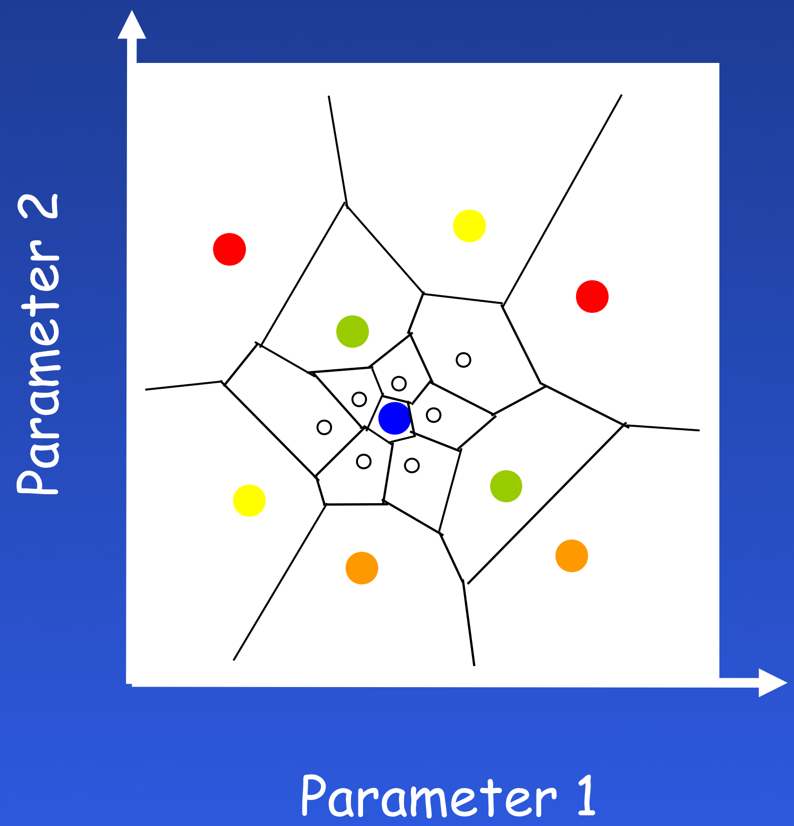


Few tuning parameters (Ns, Nr)

Based on Voronoi division of the parameter space

~ SA and GA (better according to Sambridge)

Ns new samples generated into
Nr selected cells

NA for dispersion curves (DC) inversion

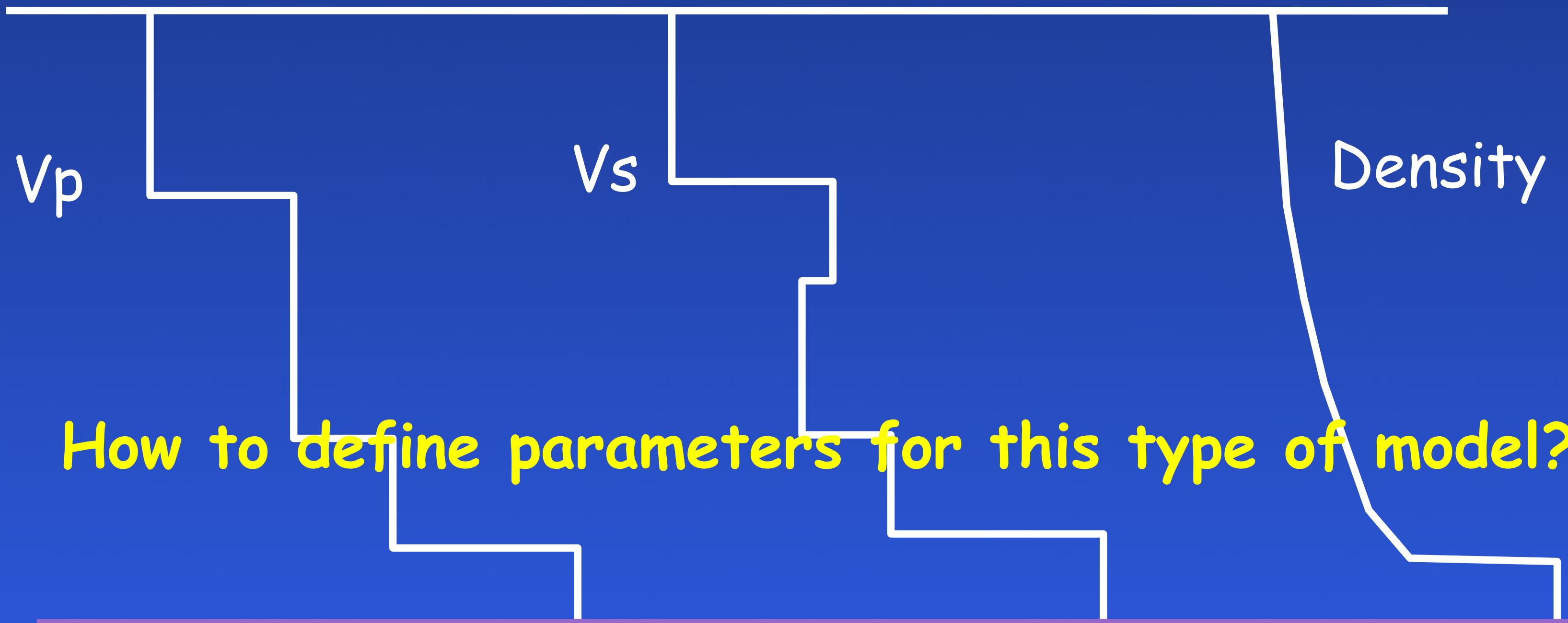
High number of forward computation required (~50,000)

Computation of DC for 1D elastic model =
numerical process not always stable

(Wathelet, 2005)

==> Battery of tests to automatically tune computation
==> Improvement of algorithm efficiency (~ ms/model)

4. Conditional parameter spaces

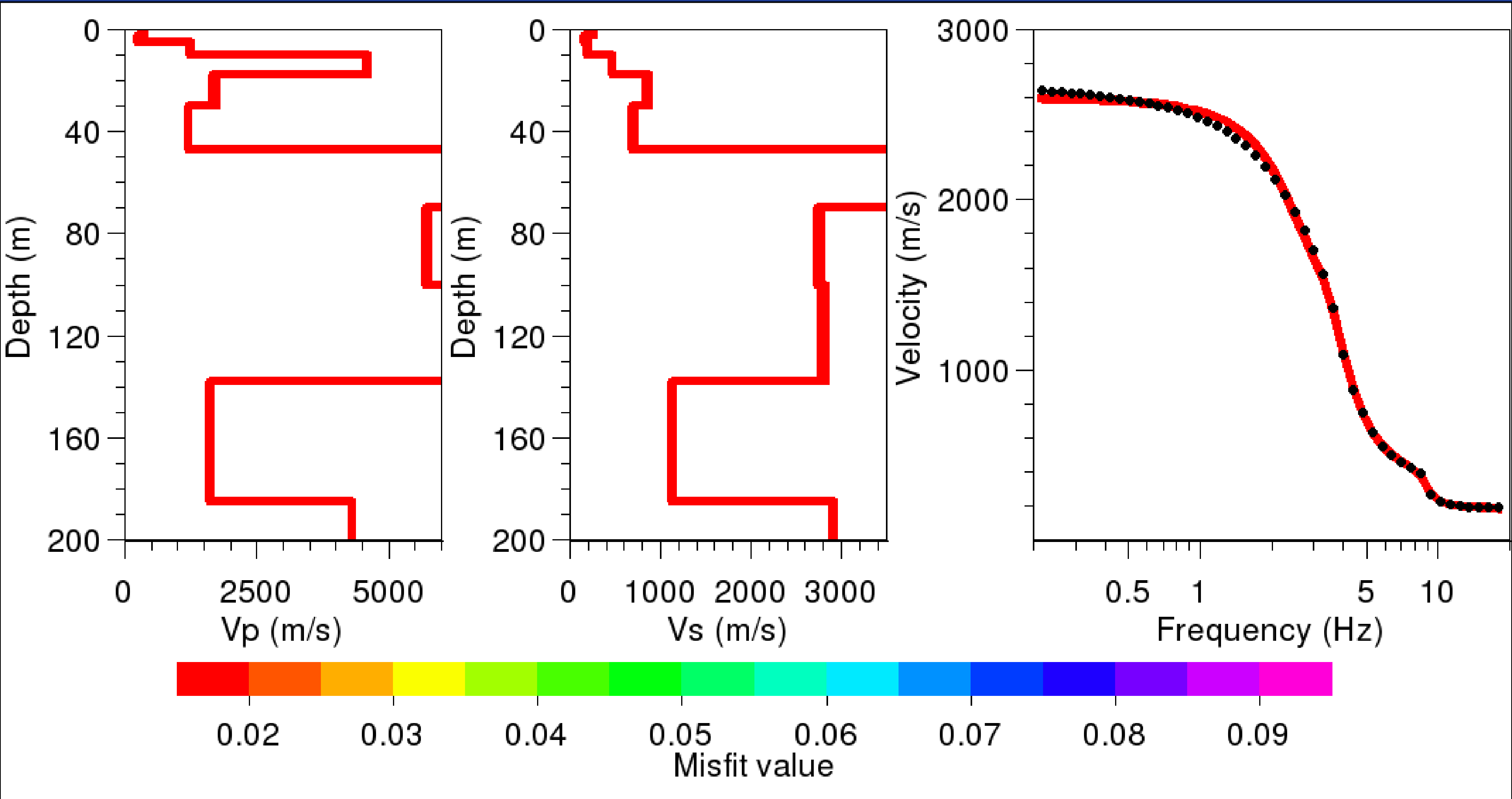


How to define parameters for this type of model?

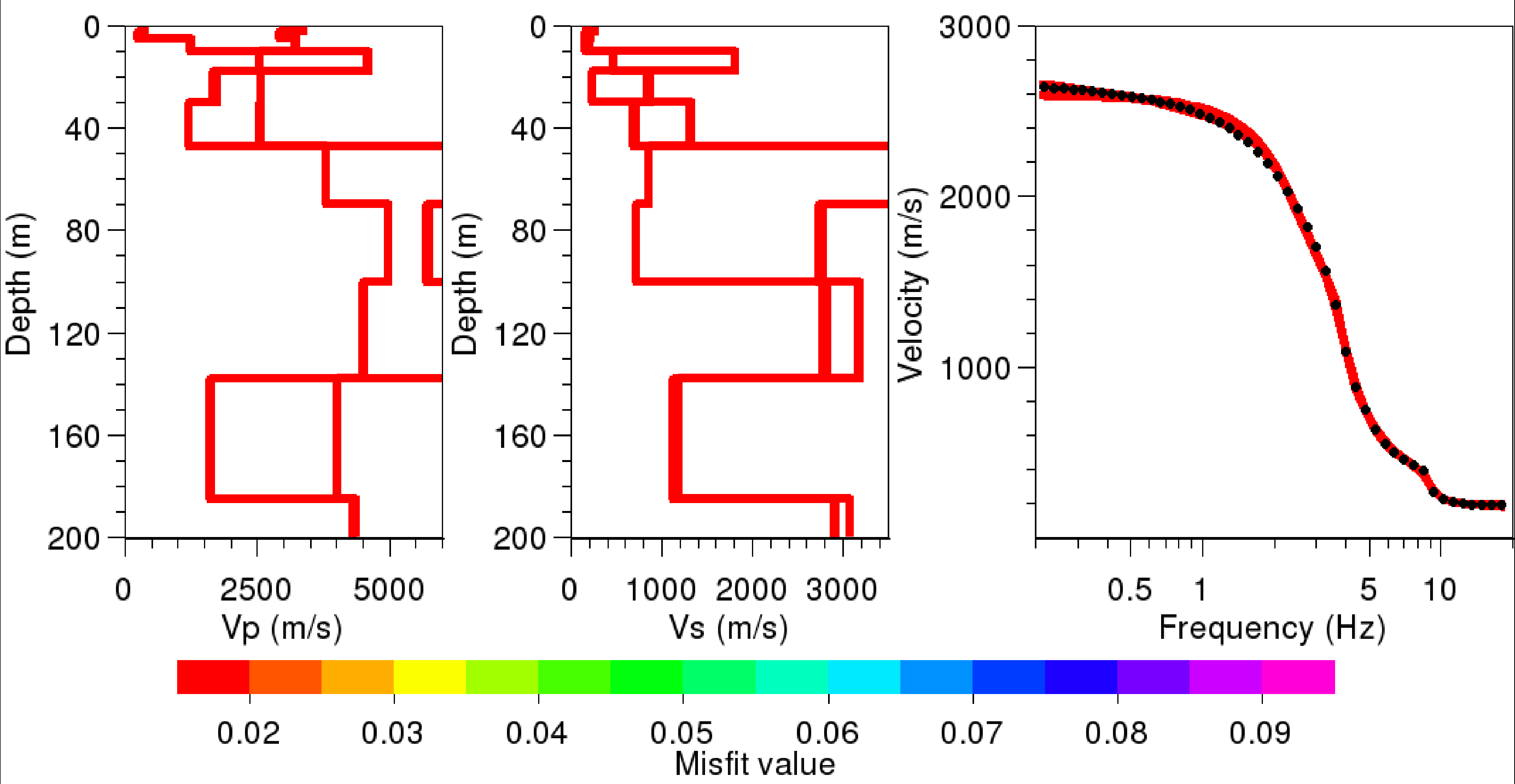
- fixed thicknesses, Poisson's ratios fixed, free Vs in each layer (classical approach in Herrmann's codes)

- free thicknesses, free Vs, free Vp, fixed density
BUT physical limits: conditions between Vs and Vp

Dangers of classical least square approach surface wave inversion



Dangers of classical least square approach surface wave inversion



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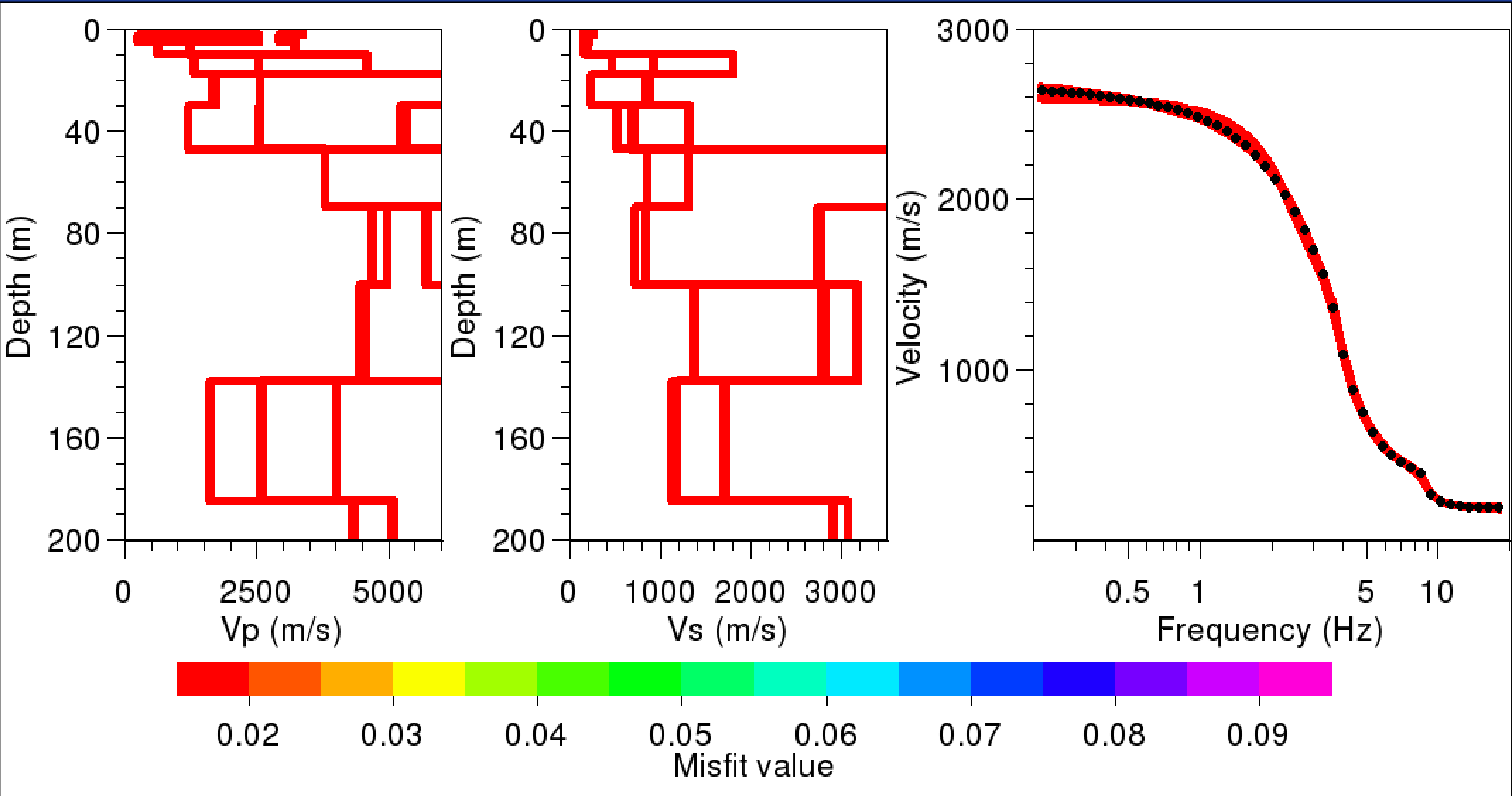

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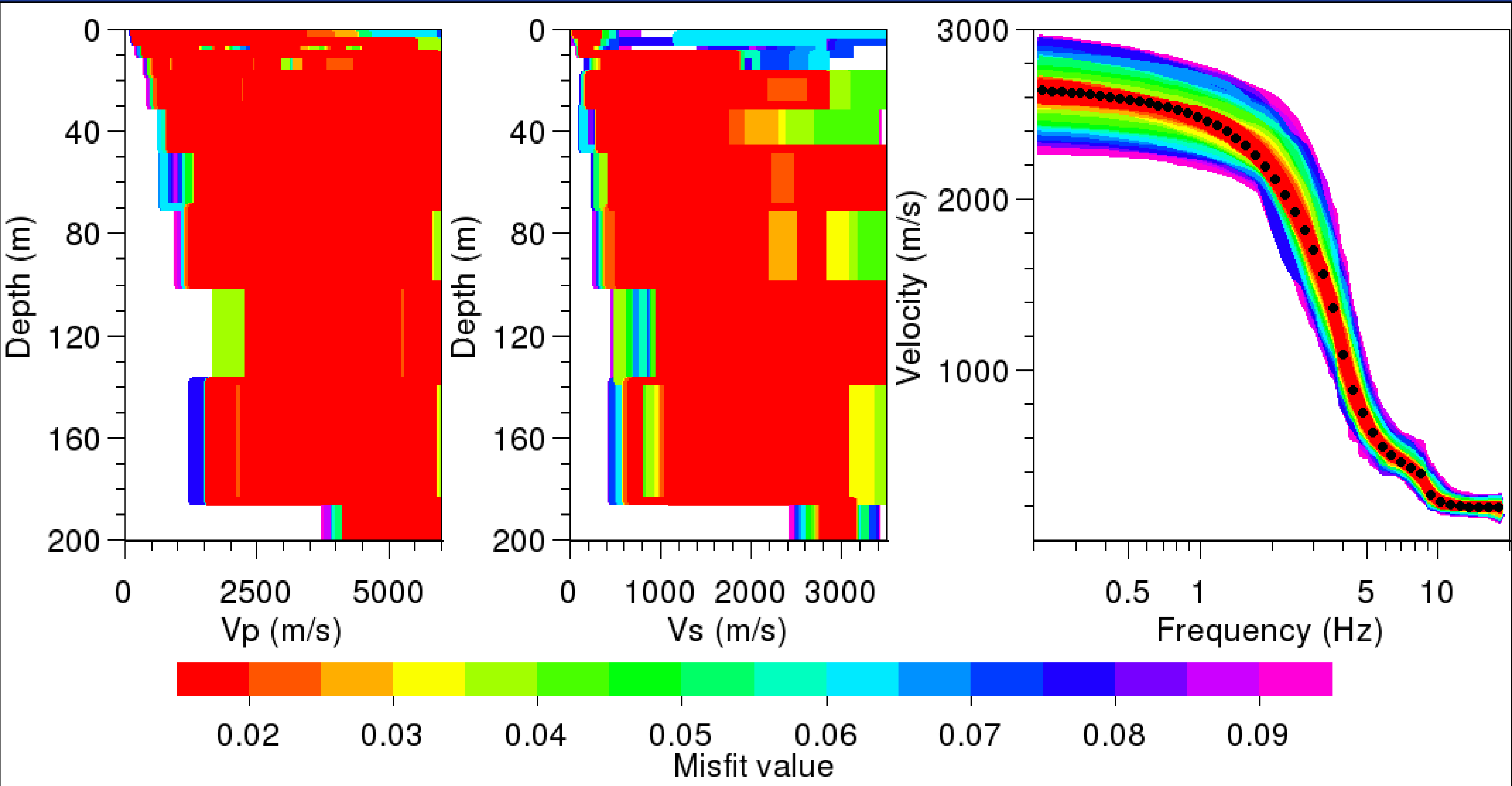
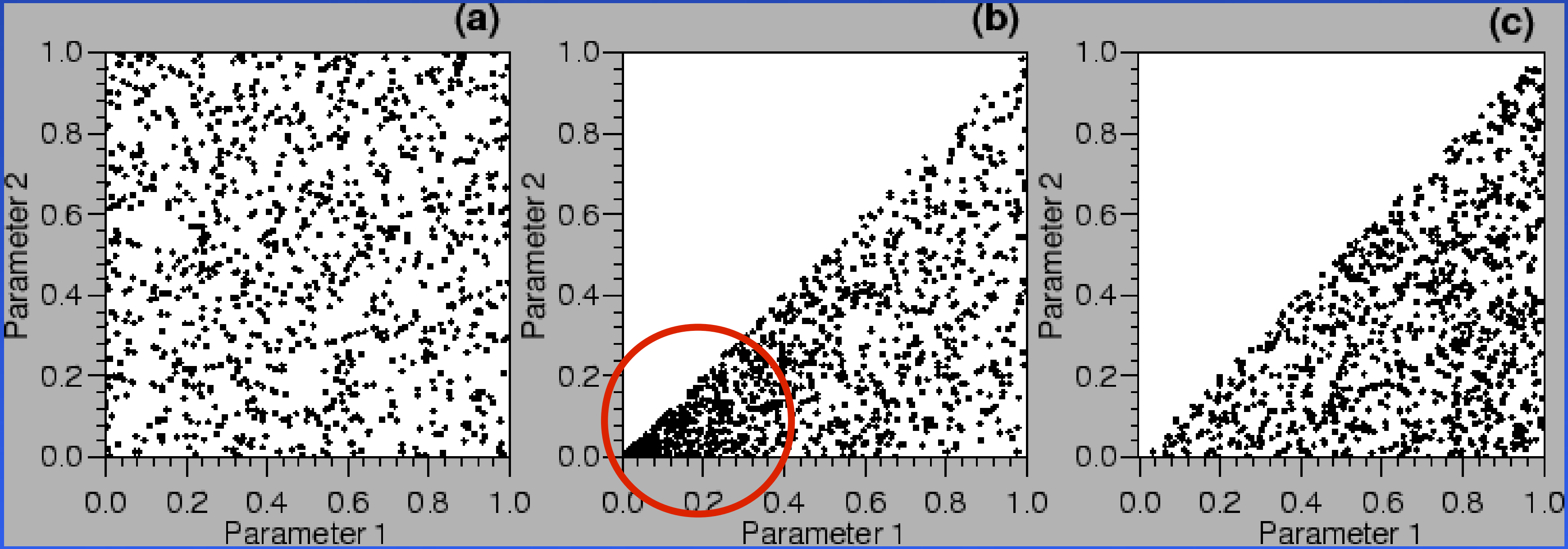


Irregular parameter space boundaries

Wathelet, M. (2008). An improved neighborhood algorithm: parameter conditions and dynamic scaling. Geophysical Research Letters, 35, doi:10.1029/2008GL033256

Sambridge: box (fixed range for all parameters)

Solution to introduce conditions: variable change

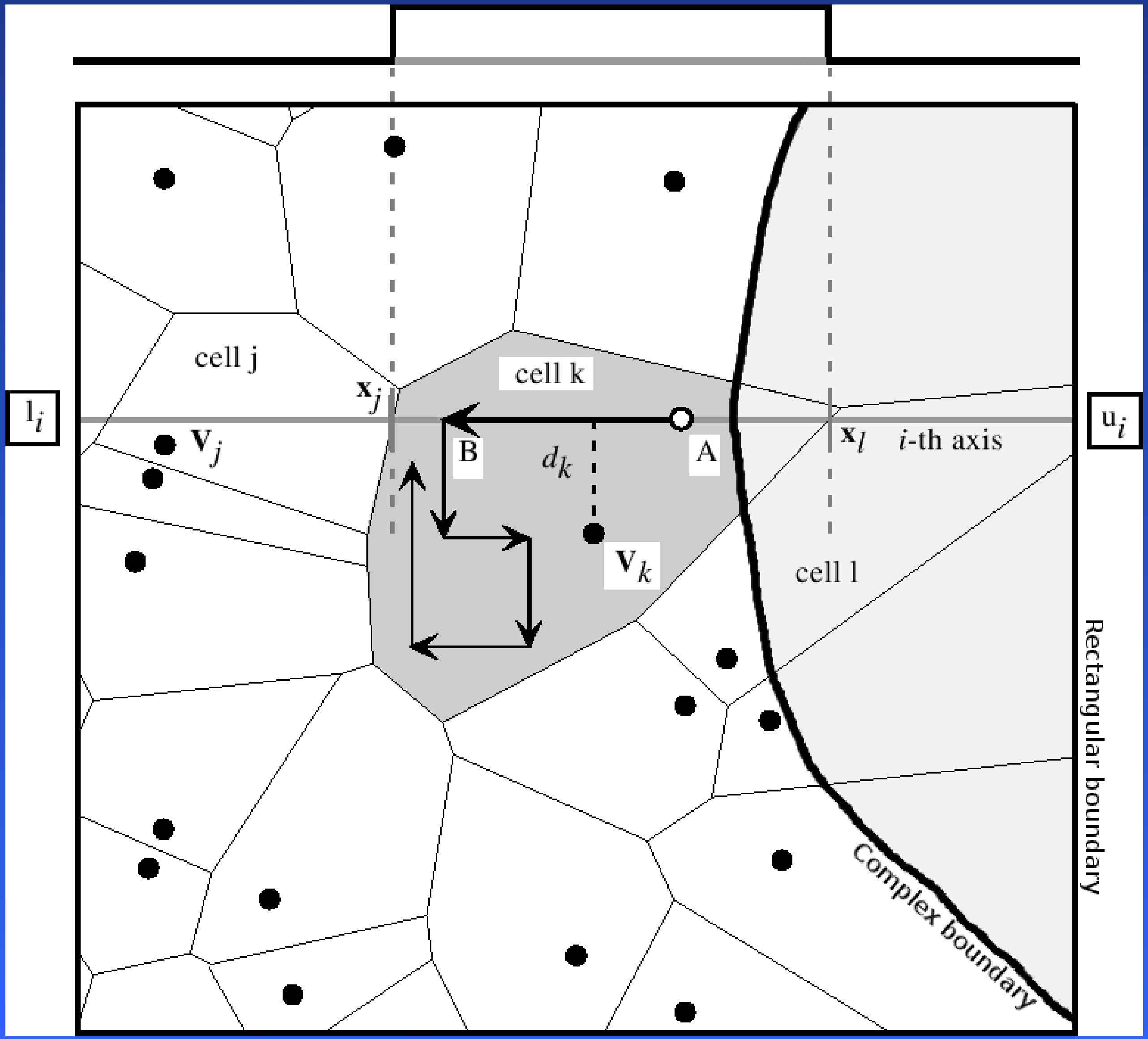


Uncontrolled Prior information

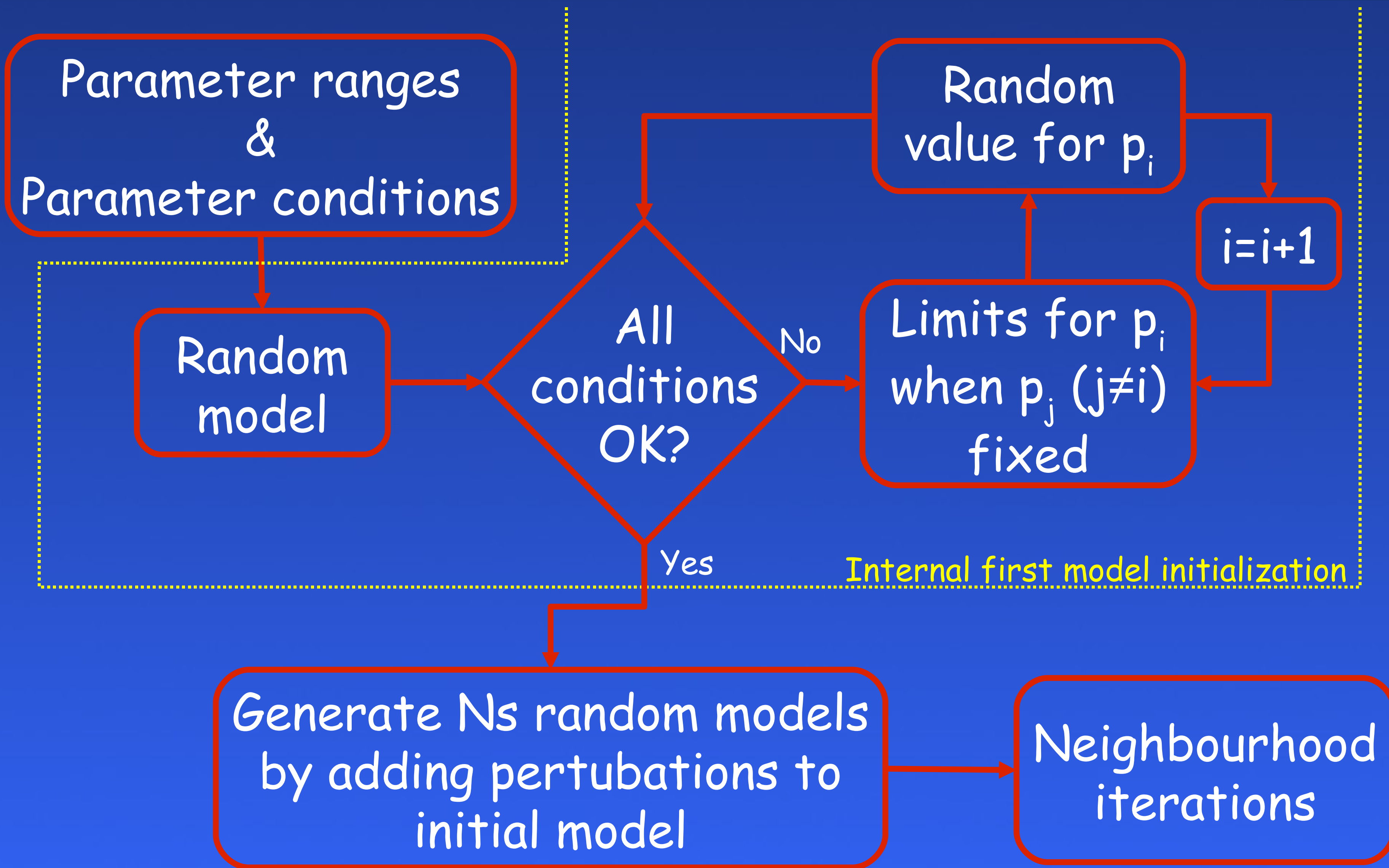
A modified Neighborhood kernel: irregular parameter boundaries

From model A
add “valid” random perturbations so that model B stays in cell k

Loop over all axes



Conditions in a Neighbourhood Algorithm



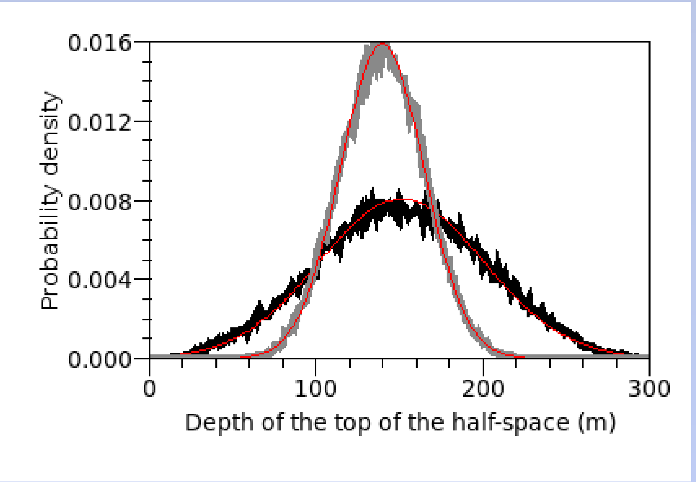
Parameterization of a ground structure

- Vp and Vs as free parameters: Poisson's ratio limitations

$$\text{Poisson's ratio} = \frac{V_s^2 - \frac{V_p^2}{2}}{V_s^2 - V_p^2}$$

Usual values for soft soil & rocks
From 0.2 to 0.5

Uncontrolled prior distribution due to a sum of parameters



- Thickness versus depth parameters

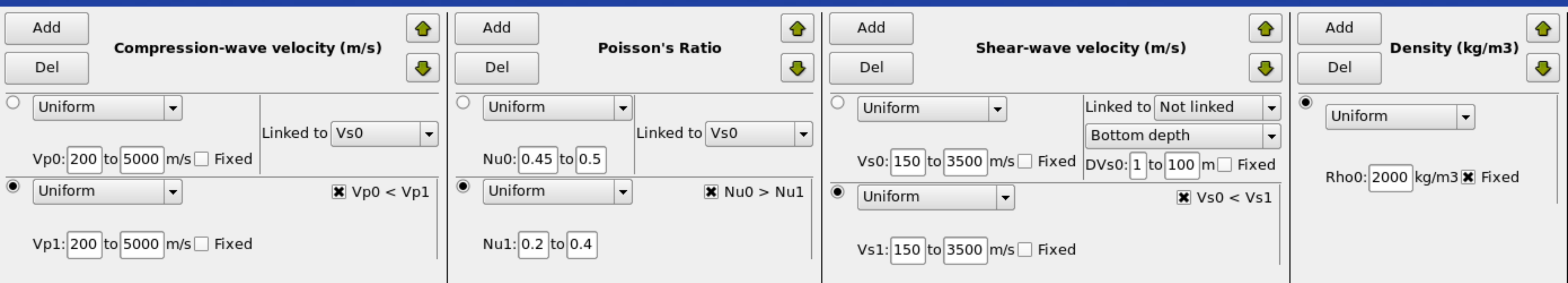
$$depth[i] > depth[i-1]$$

- Avoid Low Velocity Zones

$$V[i] > V[i-1]$$

- Parameters for non-uniform layers (gradients)
  Vt = Velocity at top
  Vb = Velocity at bottom
  Power law gradient: Vb>Vt & Vb<Vt+delta

Conversion between Vs-Vp ground model and a conditional parameter space



5 parameters

Rectangular limits

200 < TopVp0 < 5000 m/s

200 < TopVp1 < 5000 m/s

150 < TopVs0 < 3500 m/s

1 < DVs0 < 100 m

150 < TopVs1 < 3500 m/s

TopRho0=2000 kg/m3

Special limits

Poisson's ratio

TopVp1 > TopVp0

TopVs1 > TopVs0

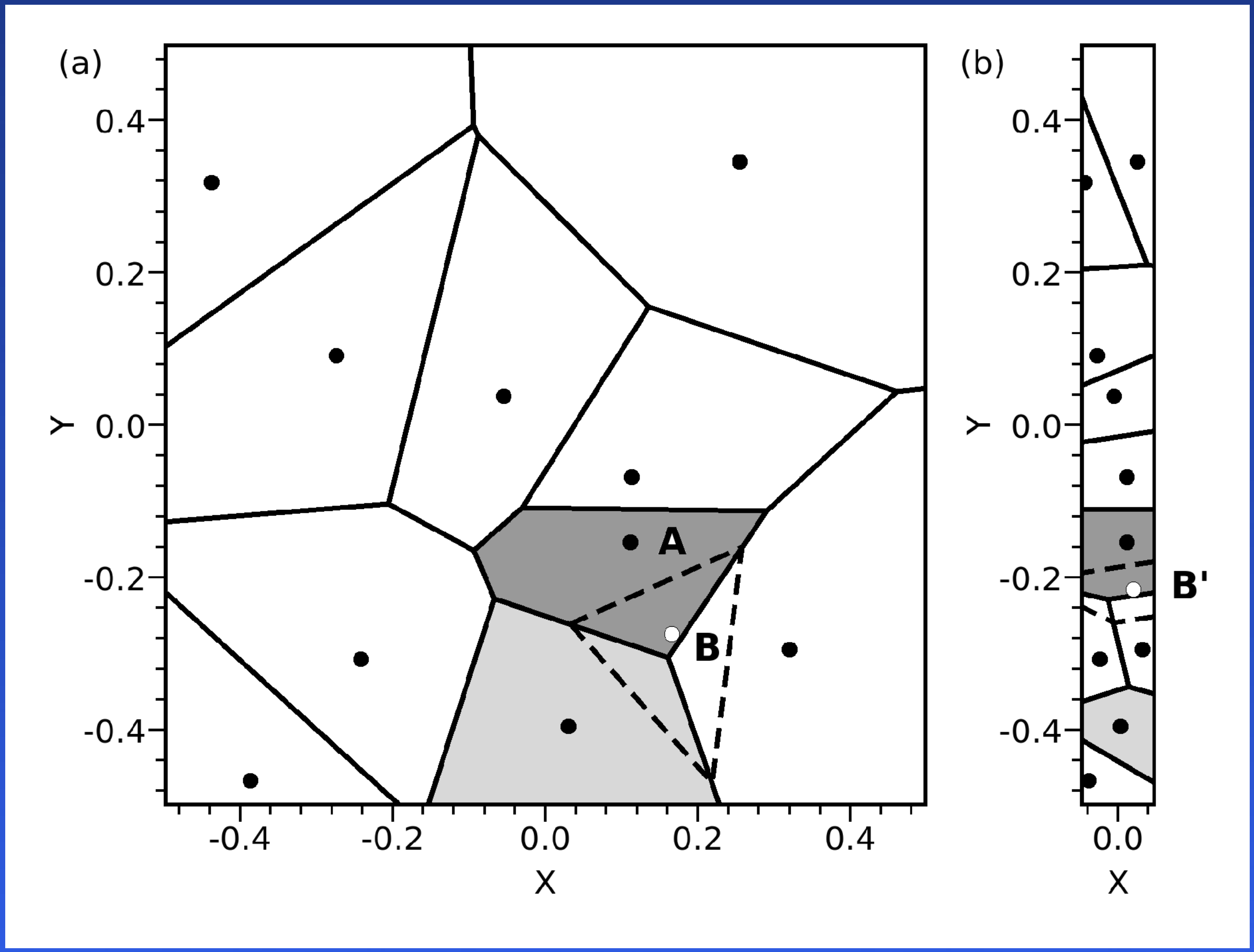
Feature summary

- Uniform or gradient layers (power law or linear)
- Fixed parameter range for prior information
- Uncorrelated Vp, Vs and density profiles
- Depth and/or thickness
- Full control over Low Velocity Zones
- Custom conditions (impedance contrast)
- Fine Poisson's ratio limits

Dynamic parameter scaling

An interesting property of Voronoi cells:

Effect of axis scaling

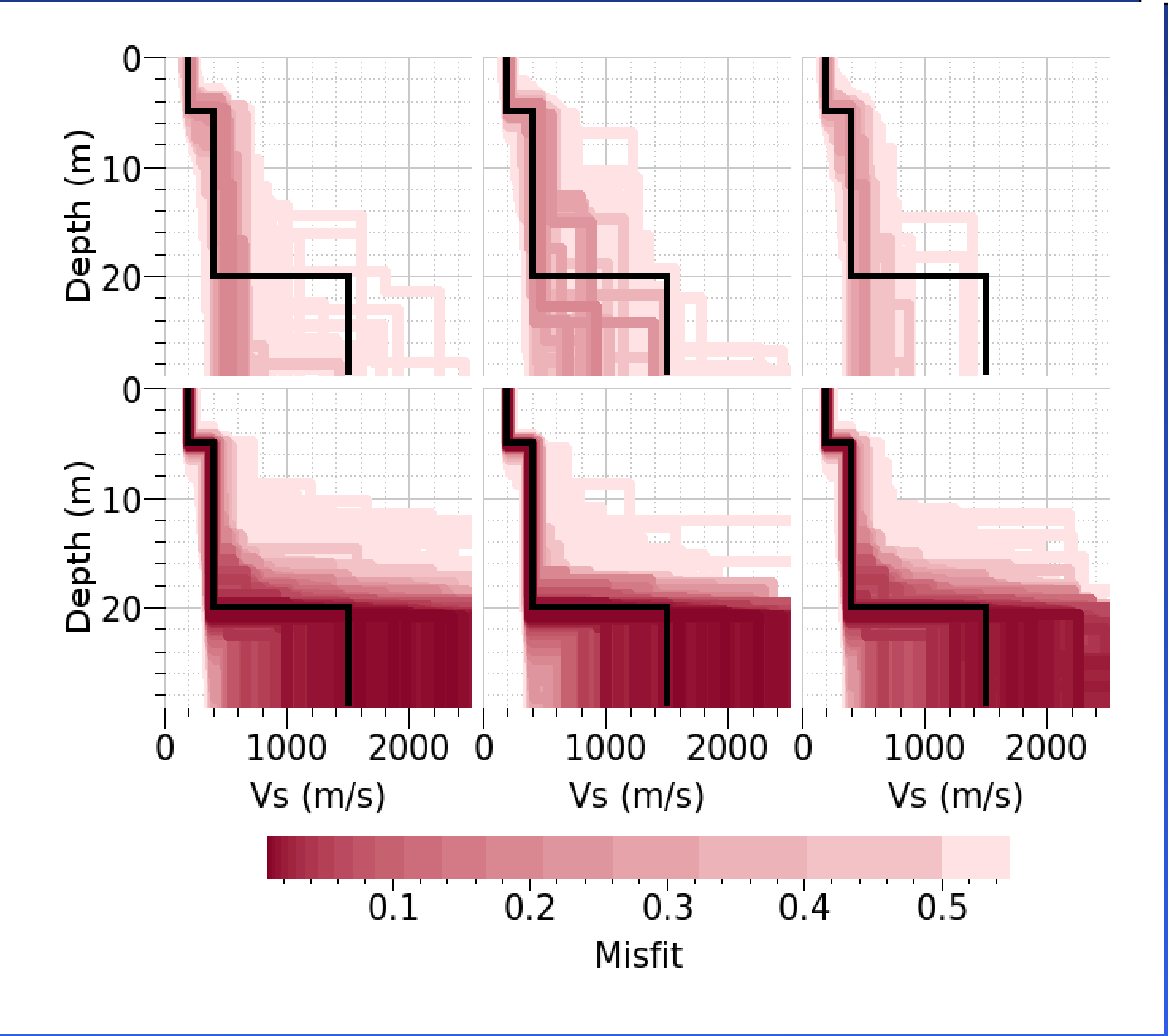


NA explores always best along the smallest axis range

Boosting exploration capabilities

Static scaling

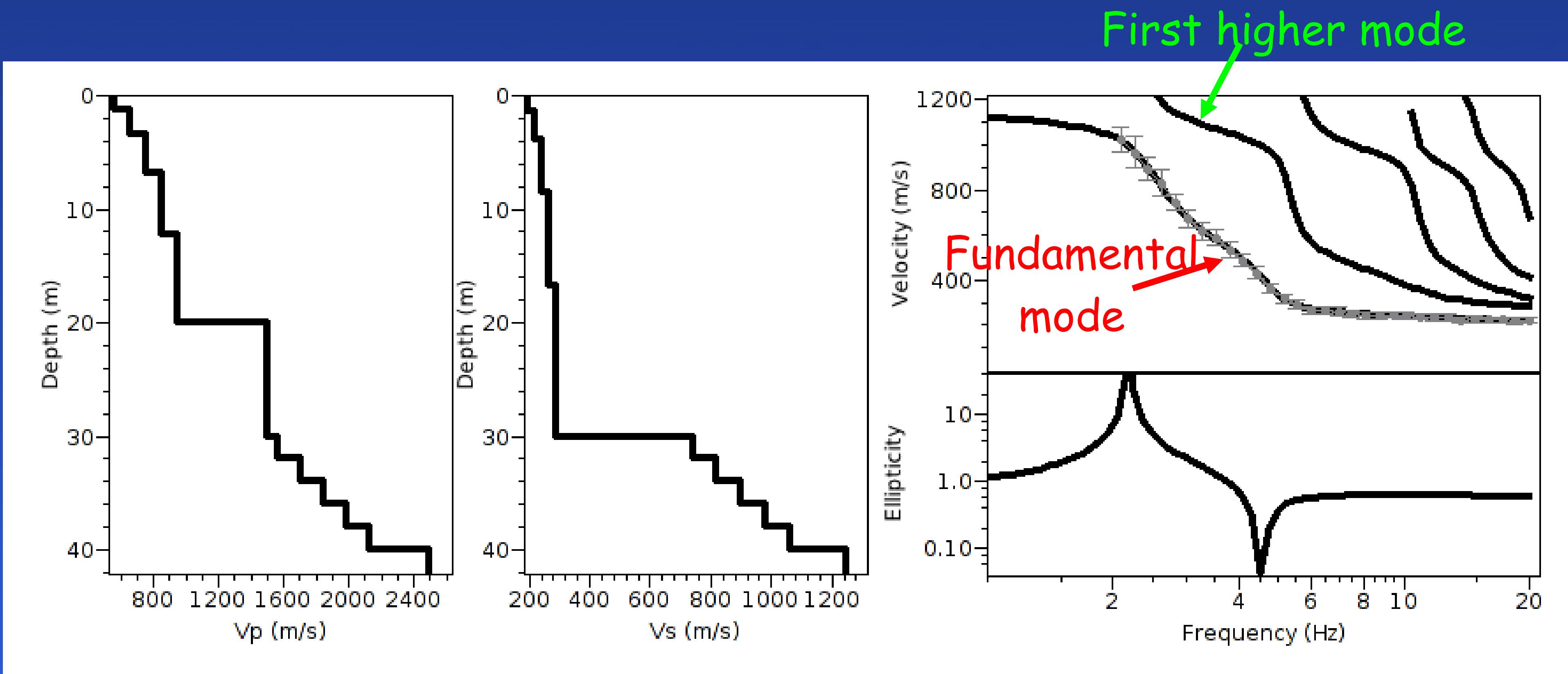
Dynamic scaling



Various random seeds: robustness

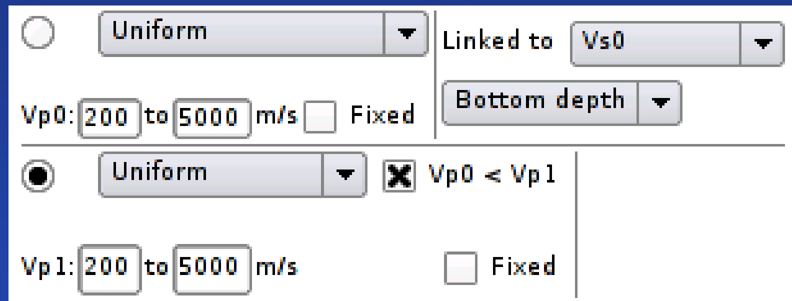
5. Dispersion curve inversion

Virtual test site: Vp and Vs structure

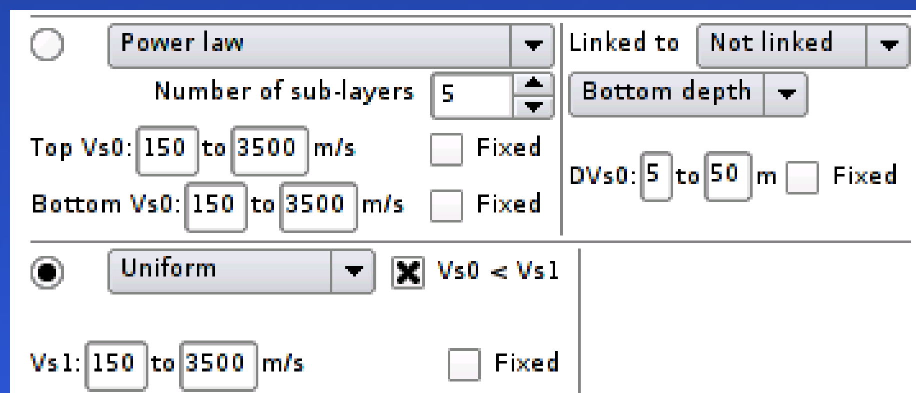


Parameterization of a 2-layer model

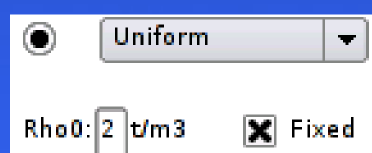
Vp



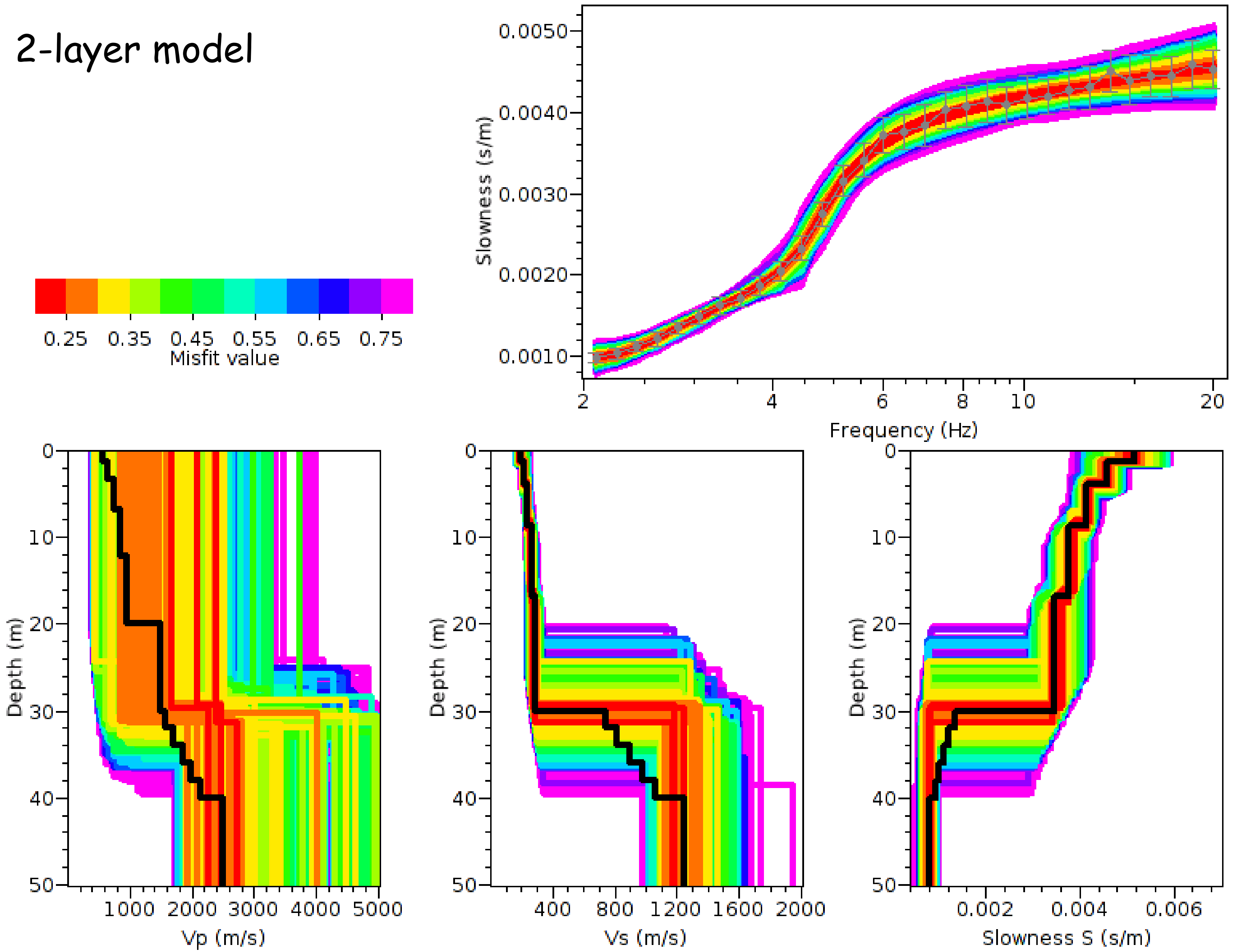
Vs



Density

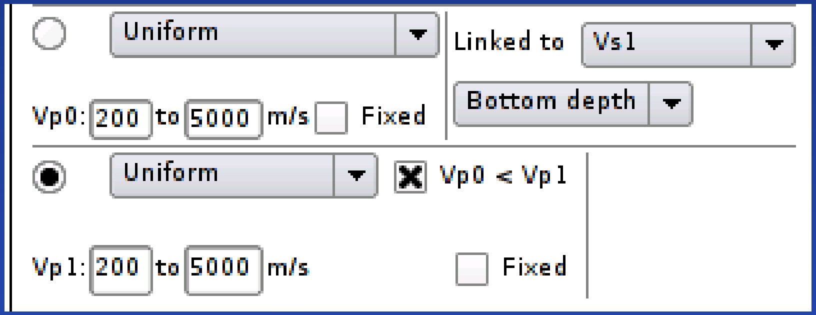


2-layer model

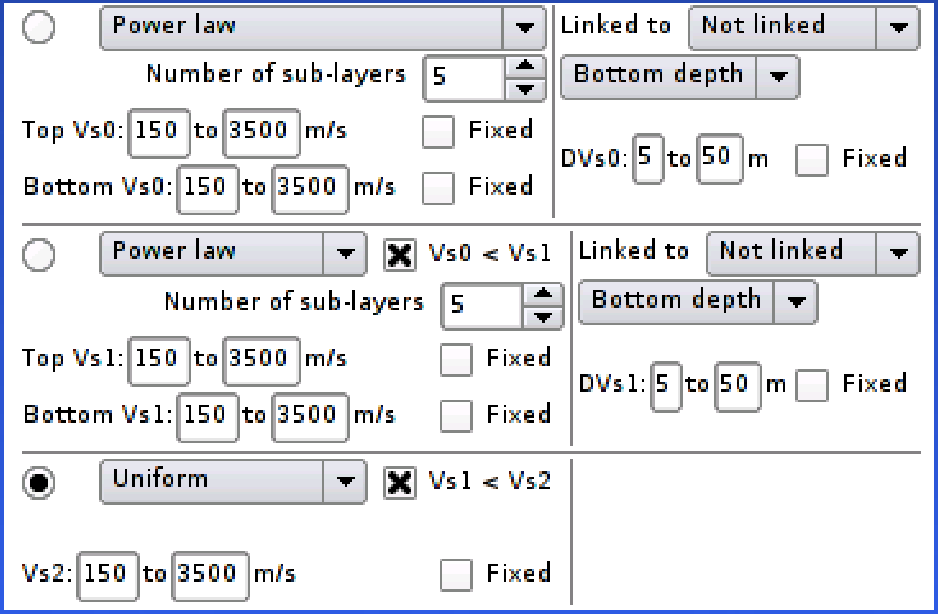


Parameterization of a 3-layer model

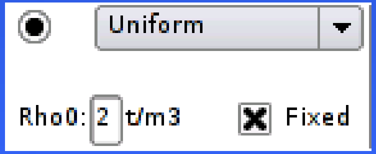
Vp



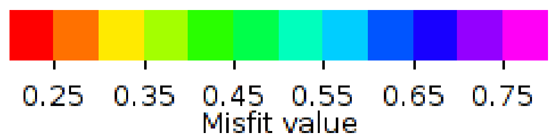
Vs

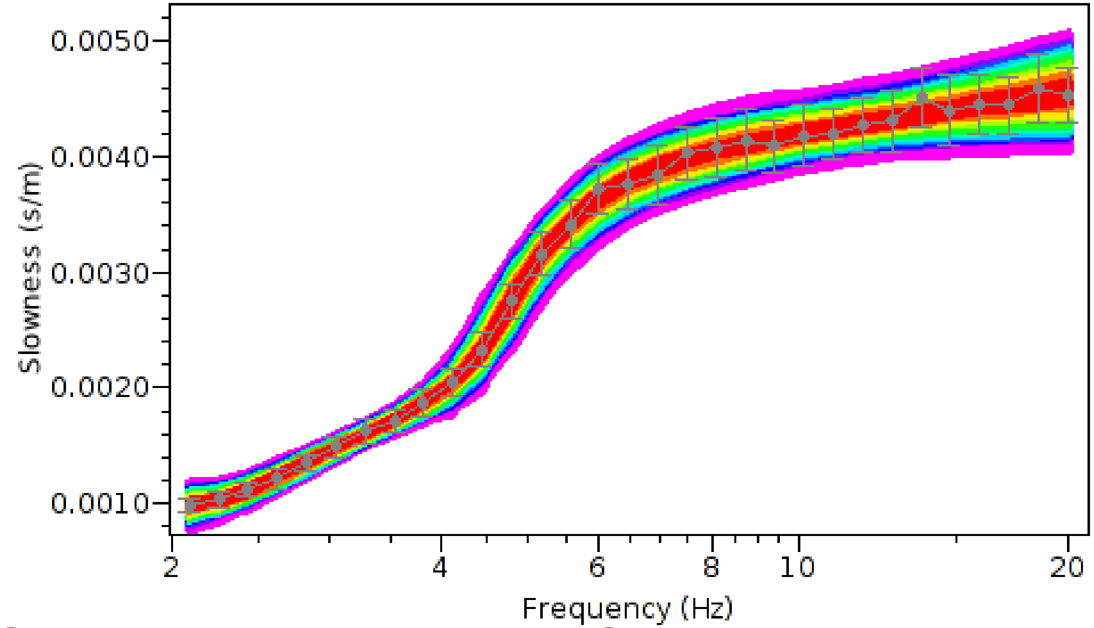


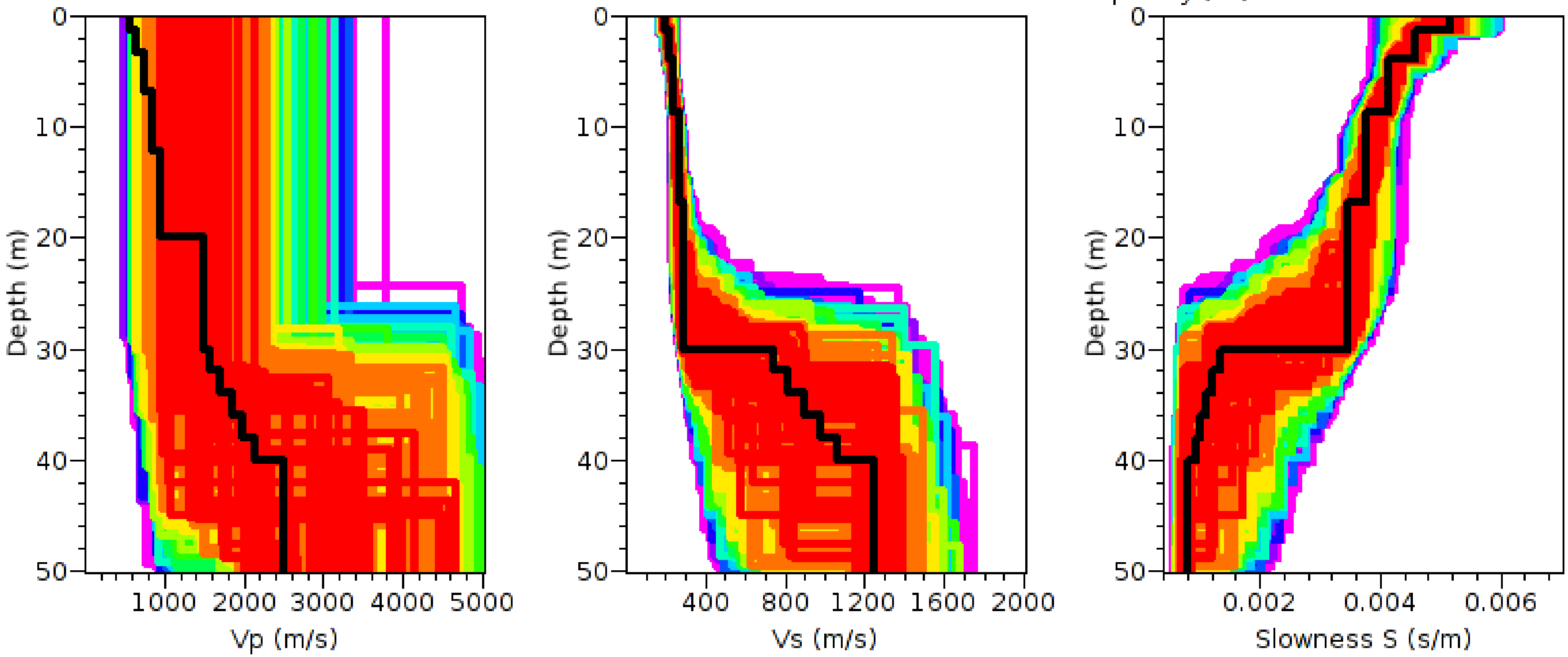
Density



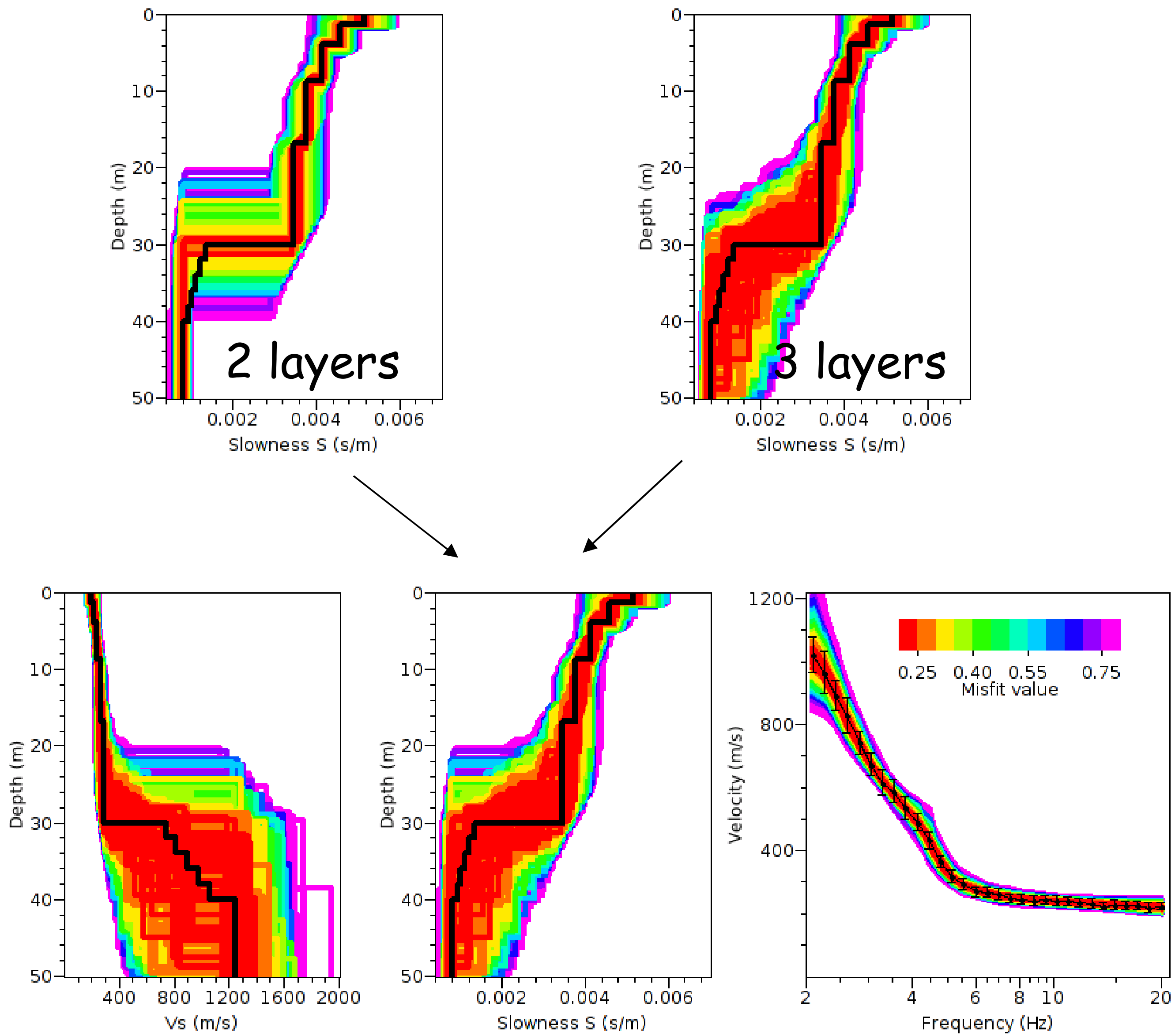
3-layer model







2-layer models or 3-layer models
What's the best solution?



We can merge all models

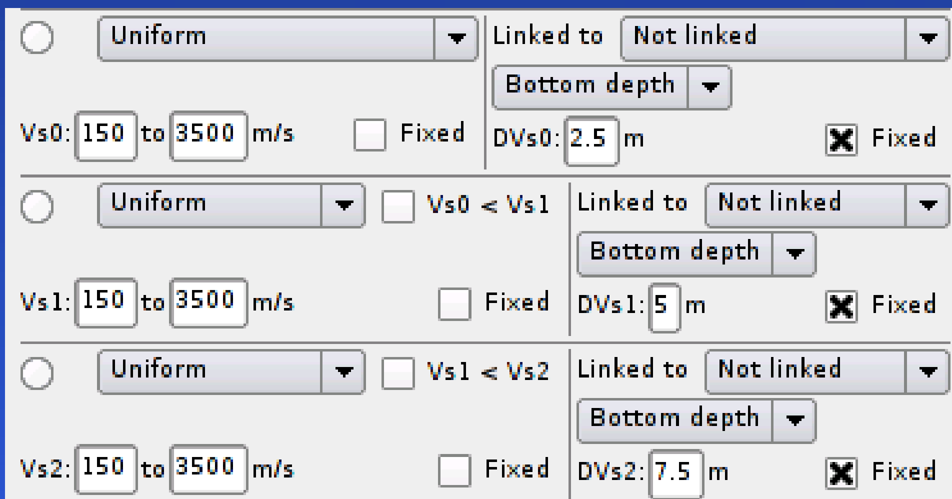
ONLY

if the misfit is computed in the same way

Parameterization of a 15-layer model

=> Identical to the classical approach
(Herrmann, linerization, gradient methods)

Vs



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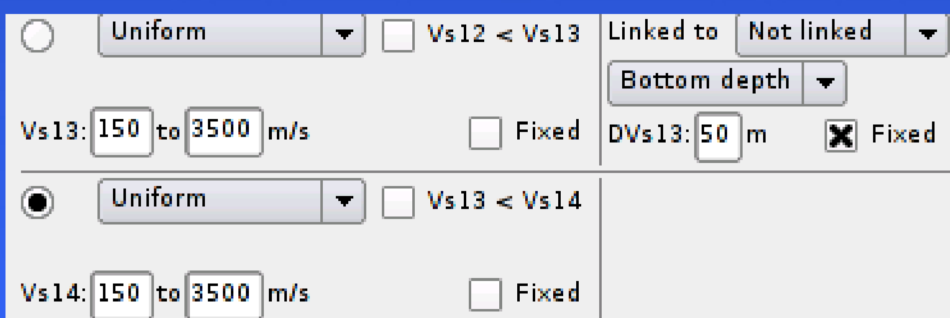
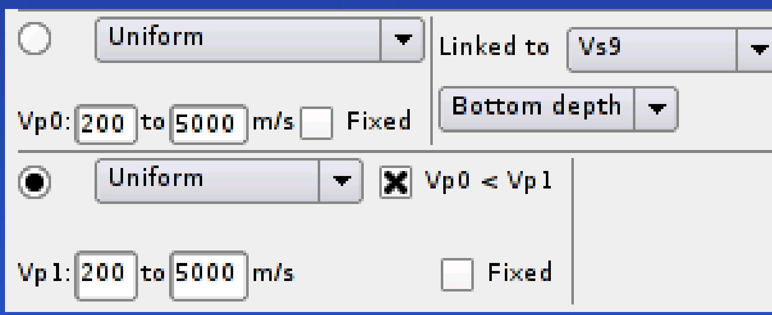
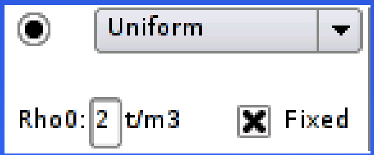


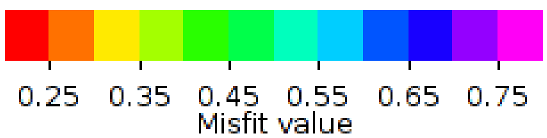
Vp

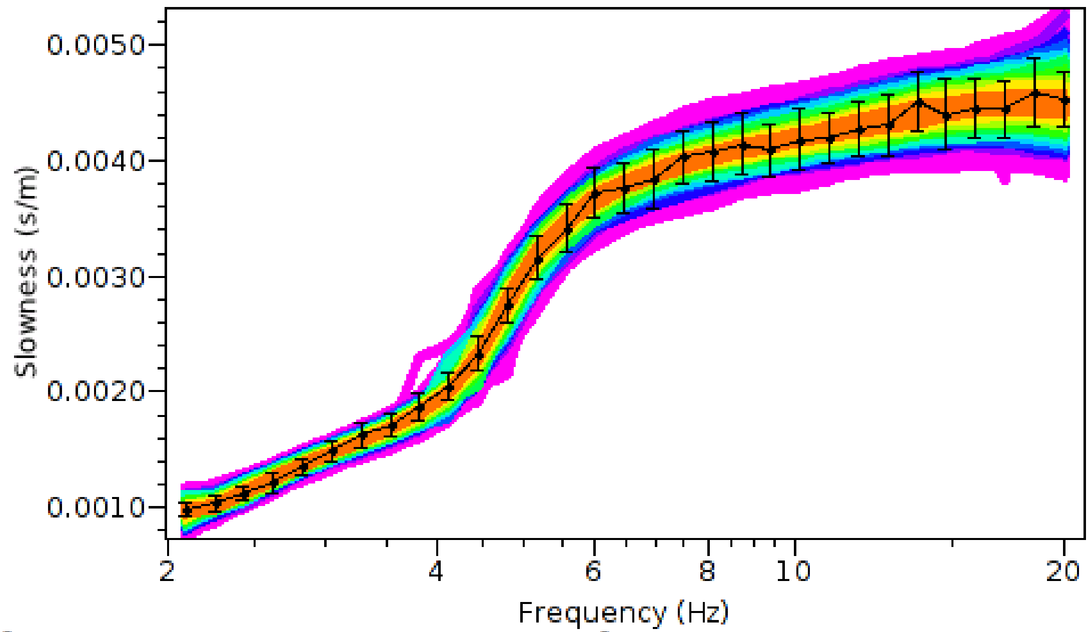


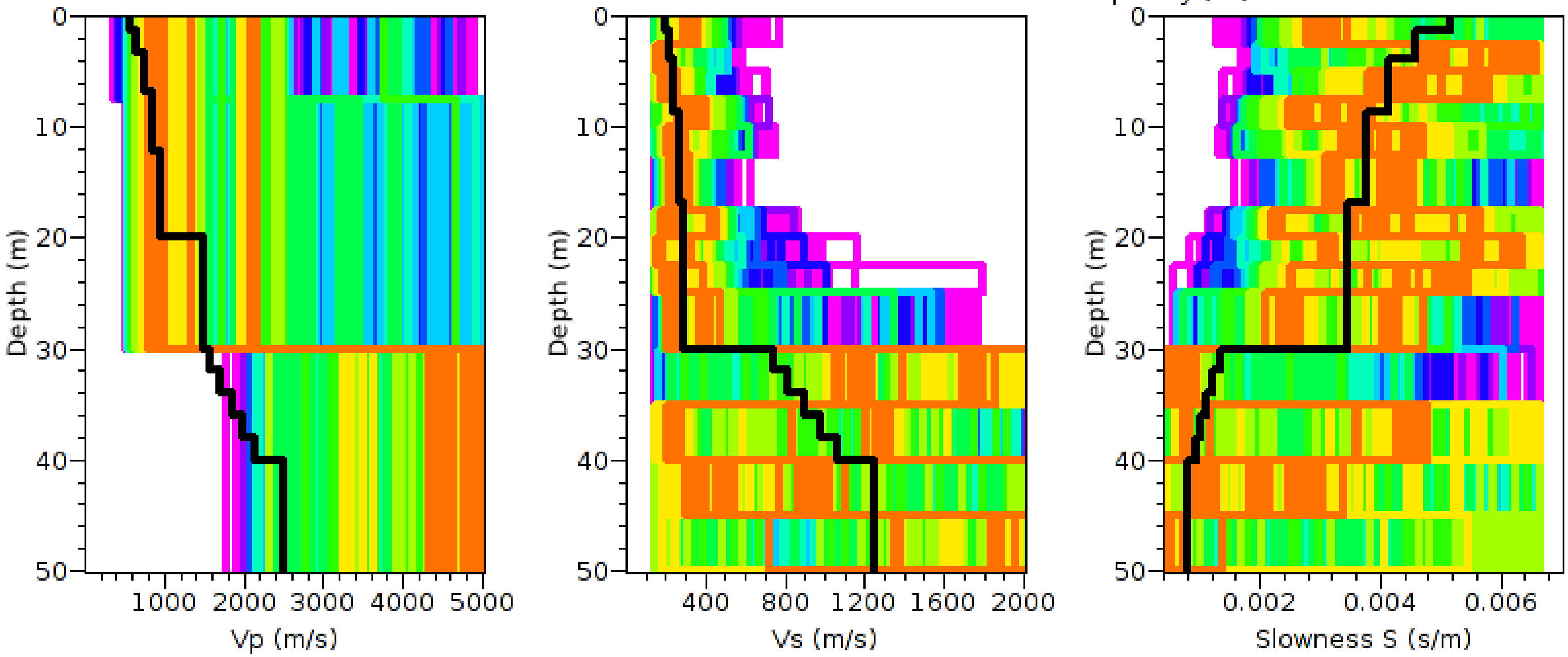
Density



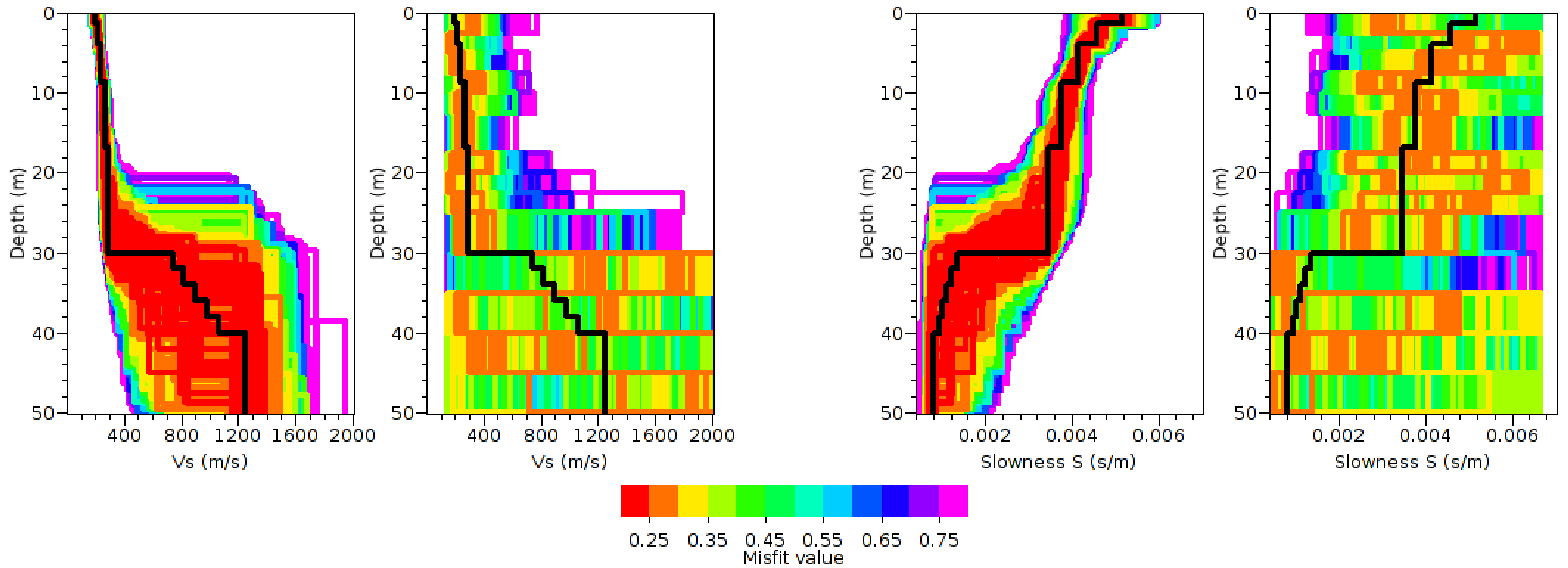
15-layer model







Parameterization of a 15-layer model versus Limited number of layers (2-3)



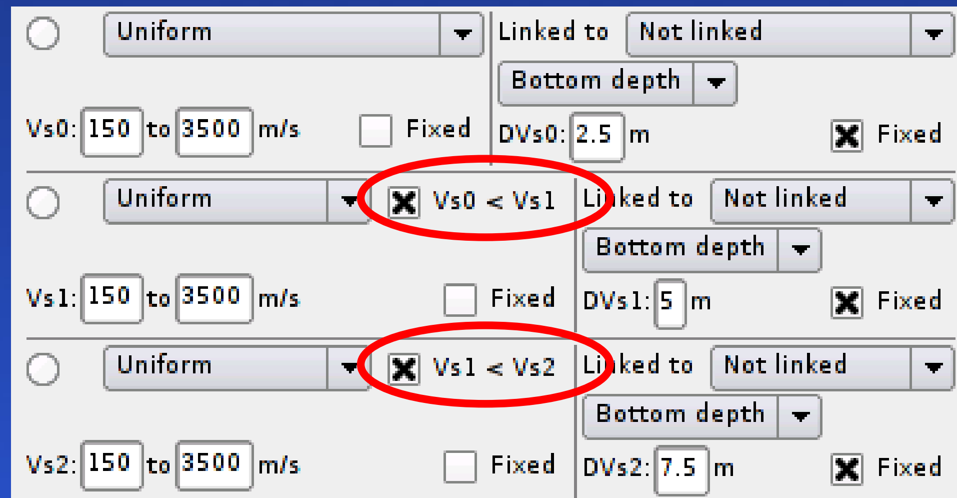
Velocity

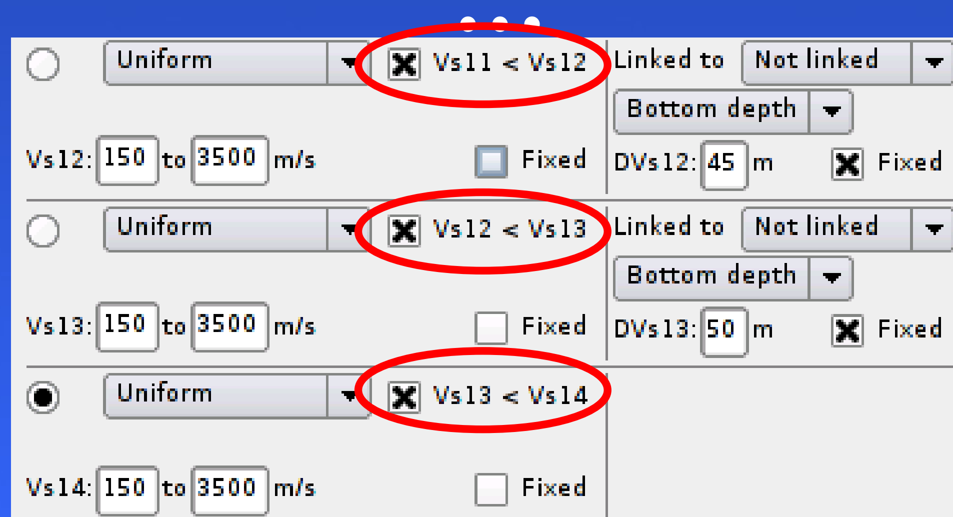
Slowness

Parameterization of a 15-layer model

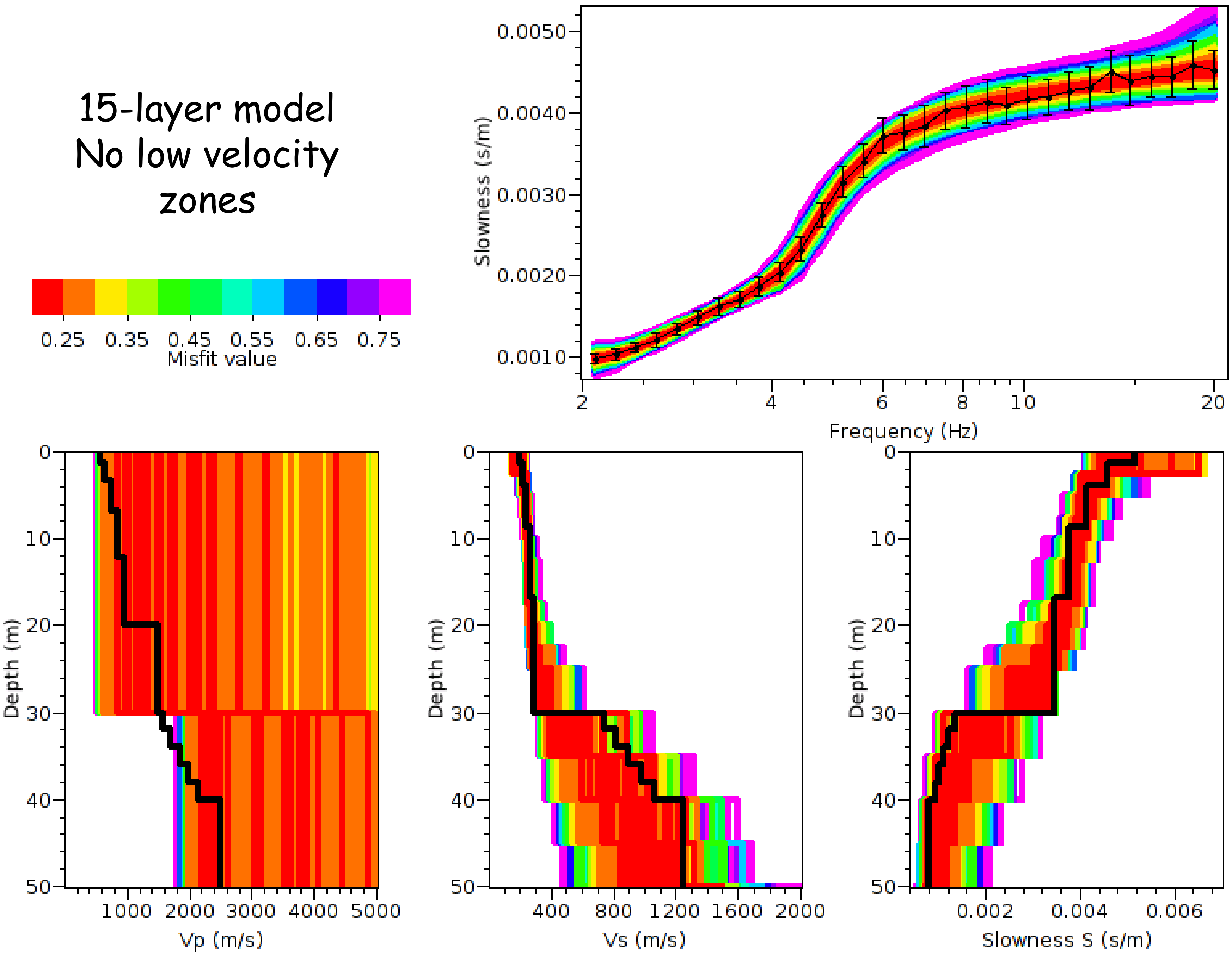
Controlling the presence of low velocity zones

Vs



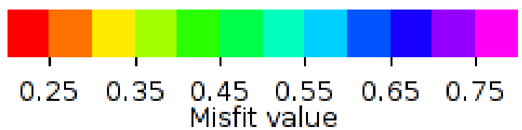


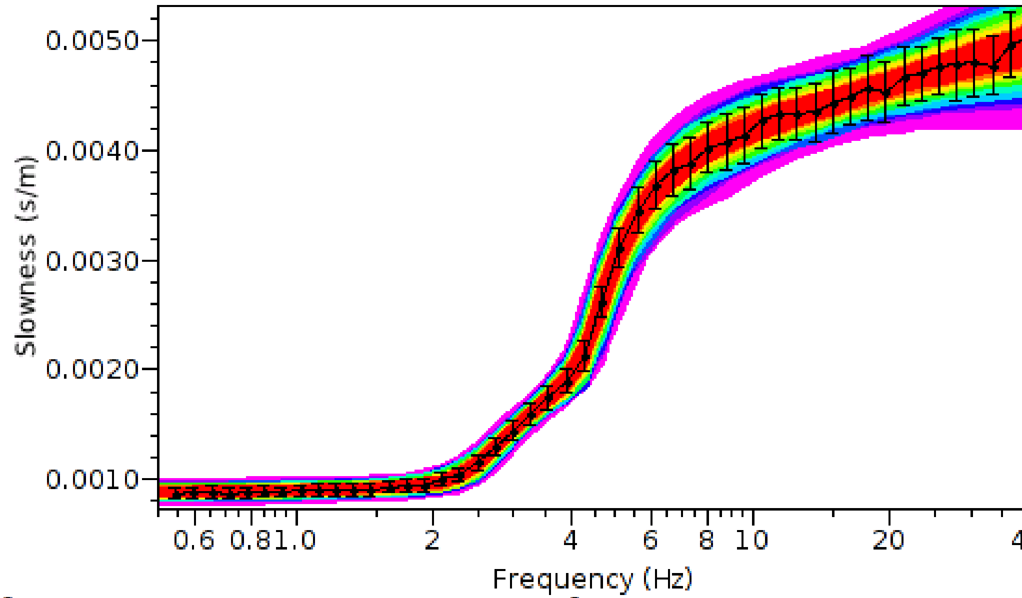
15-layer model
No low velocity zones

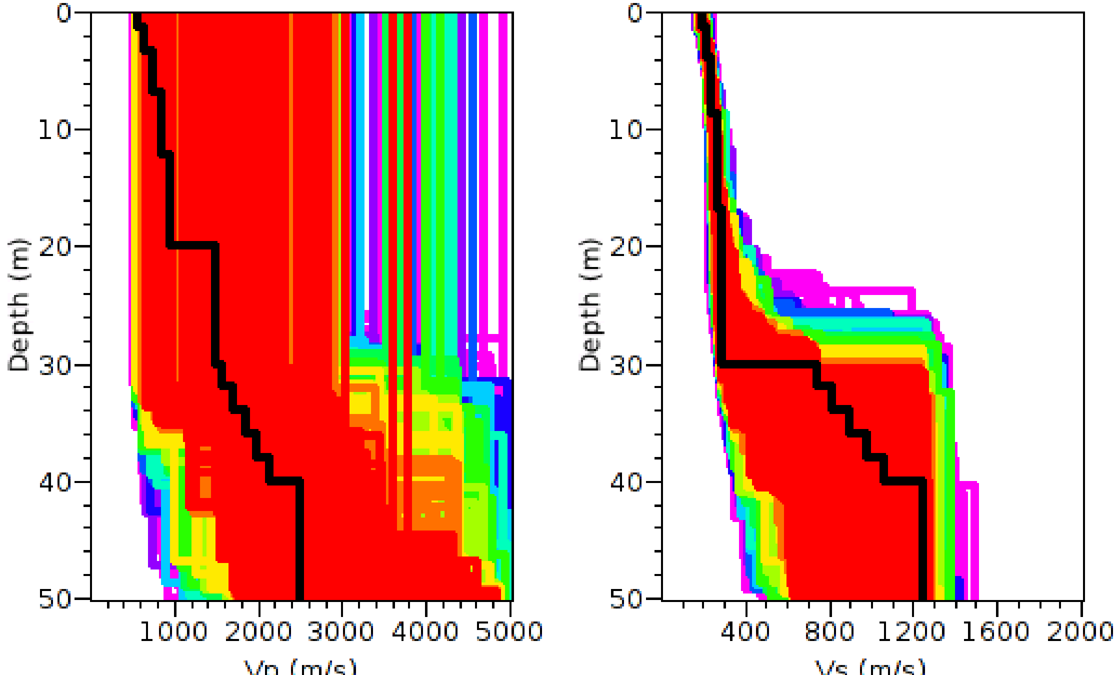


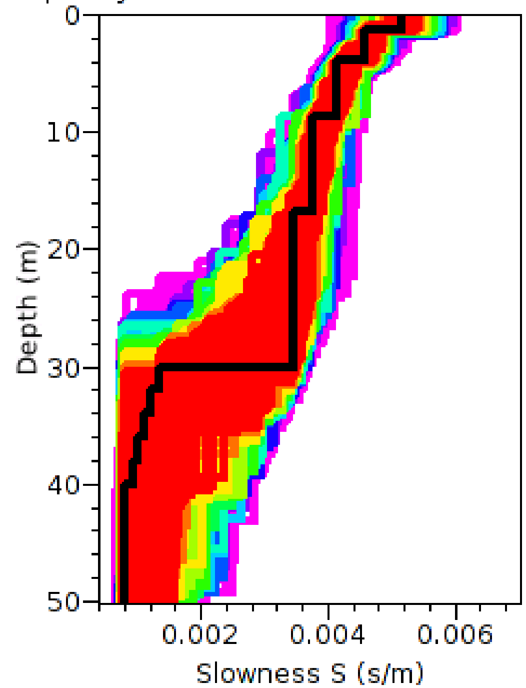
Effects of frequency range

Full range dispersion curve



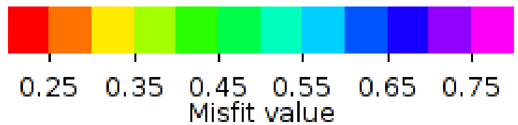


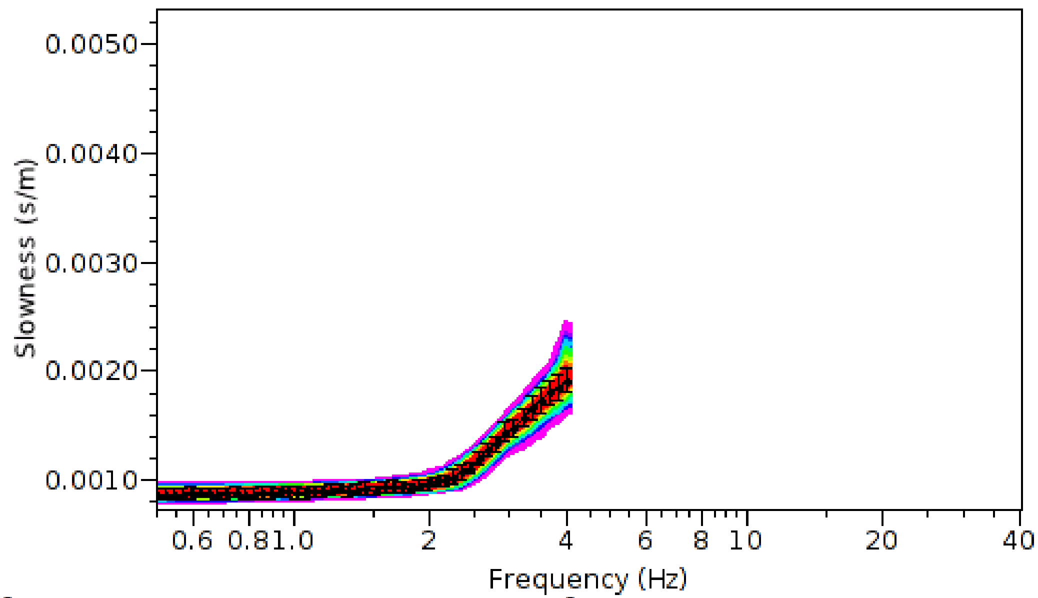


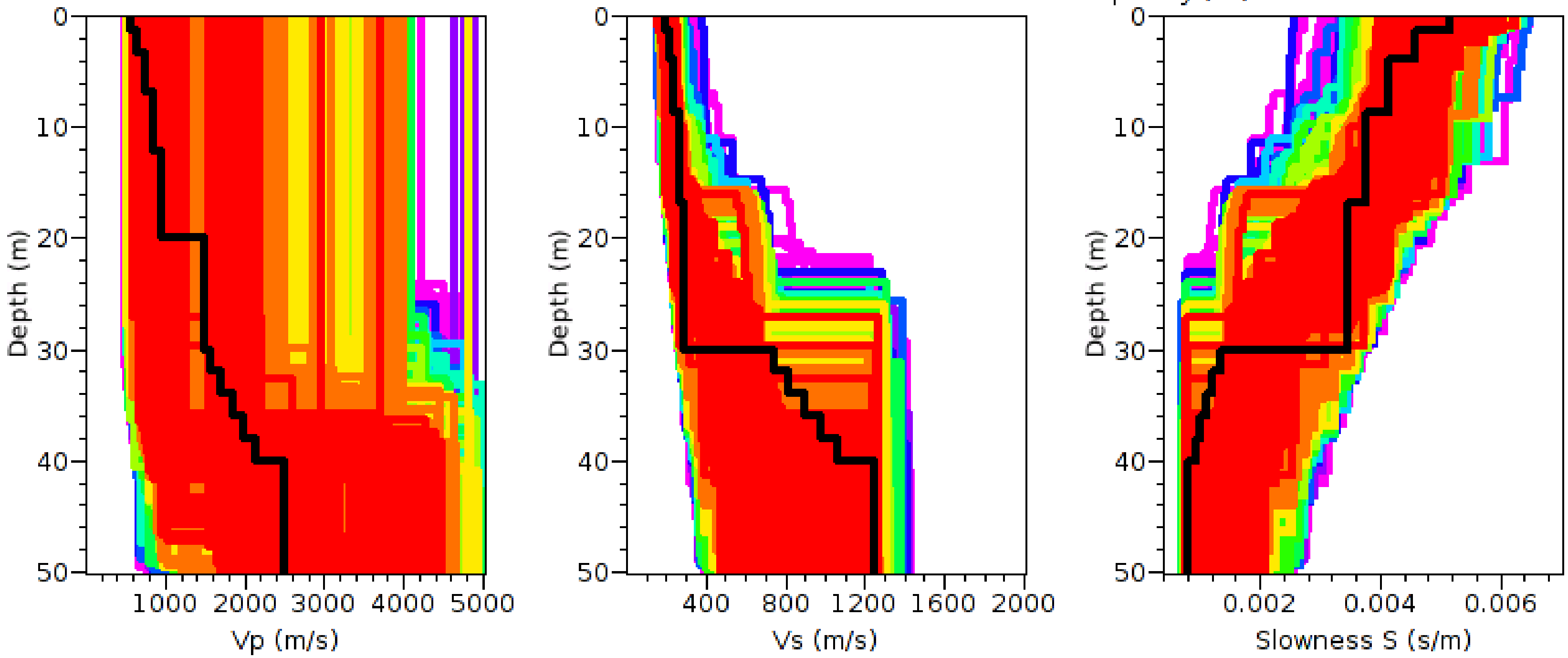


Effects of frequency range

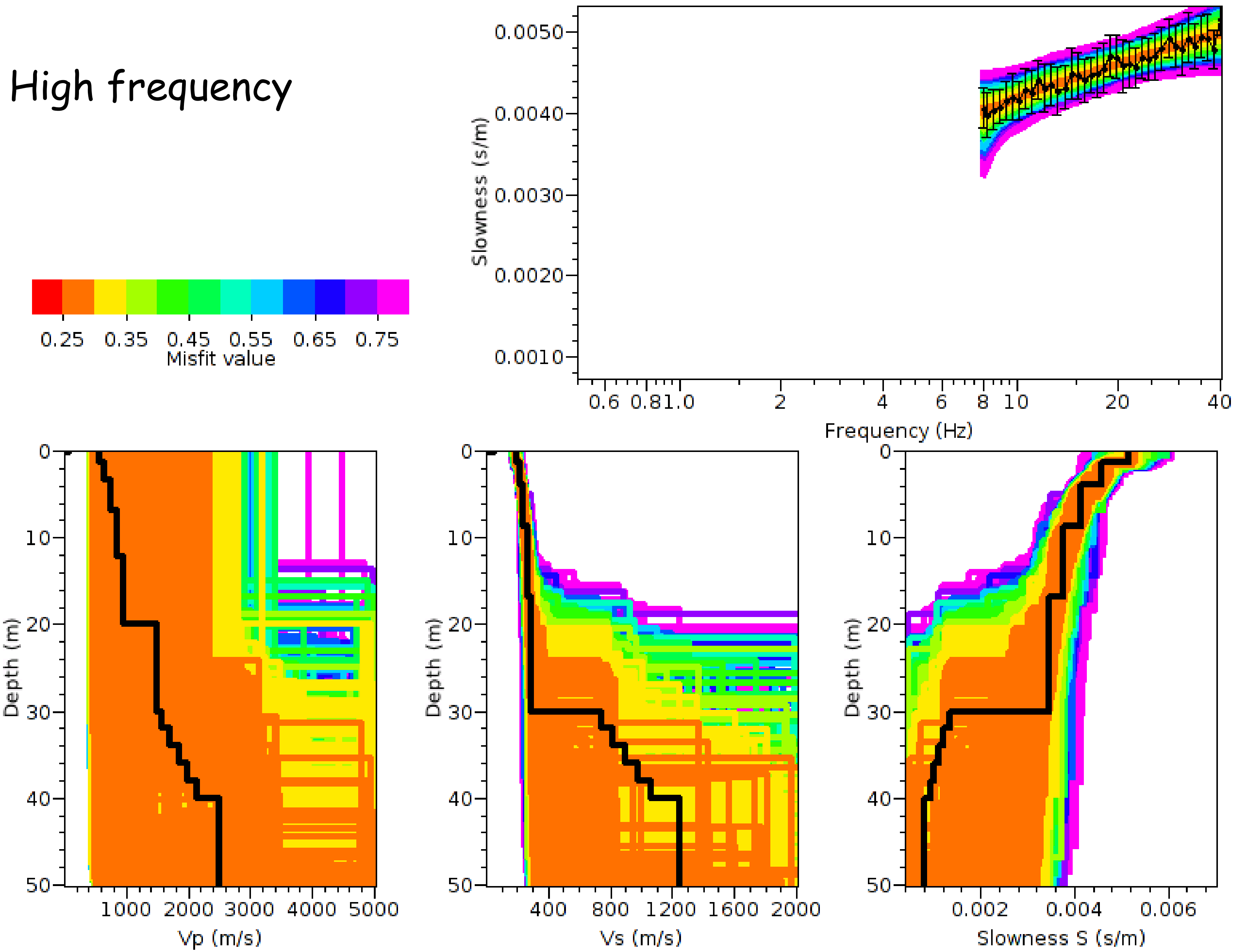
Low frequency







High frequency



Effects of frequency range

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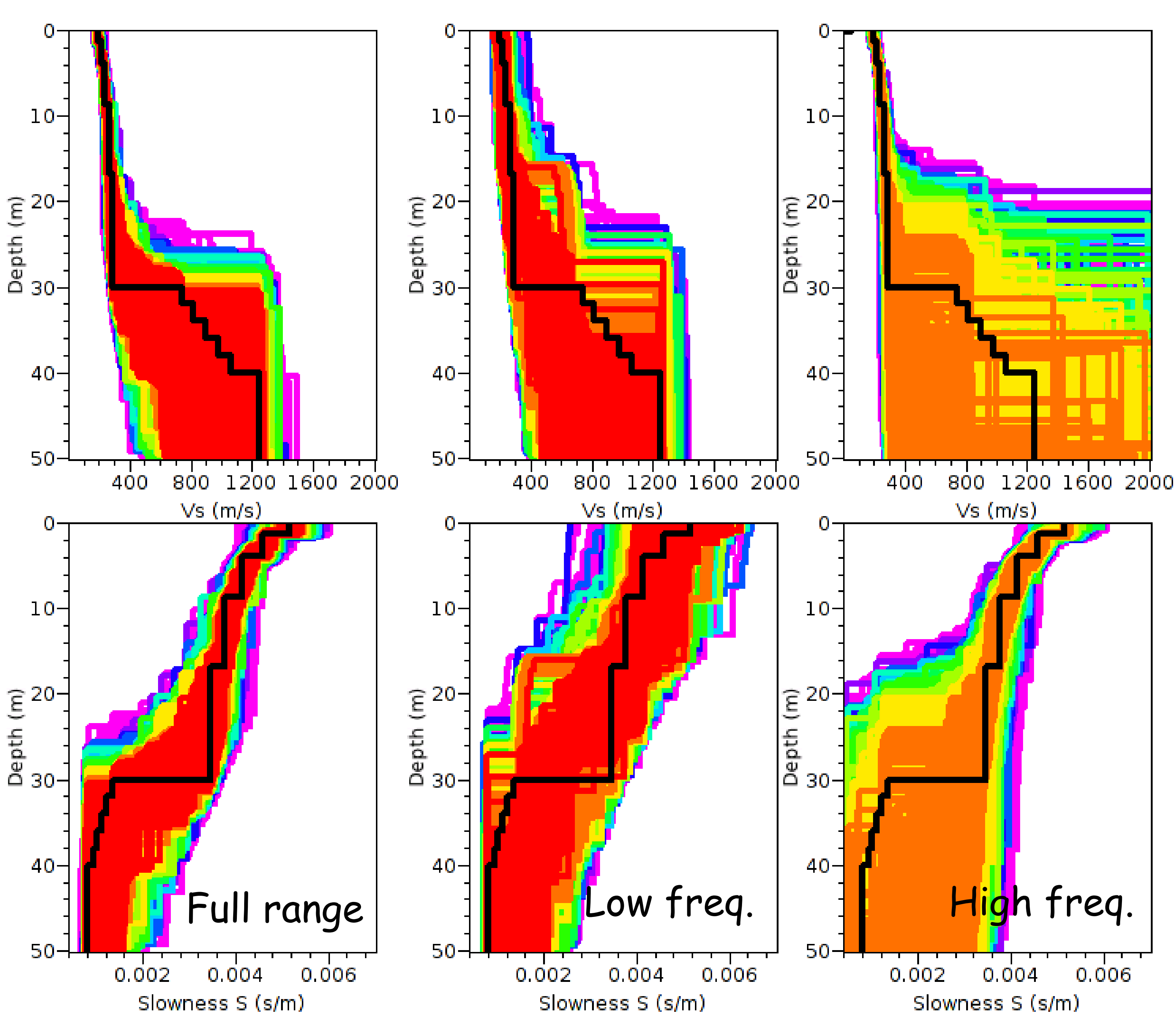
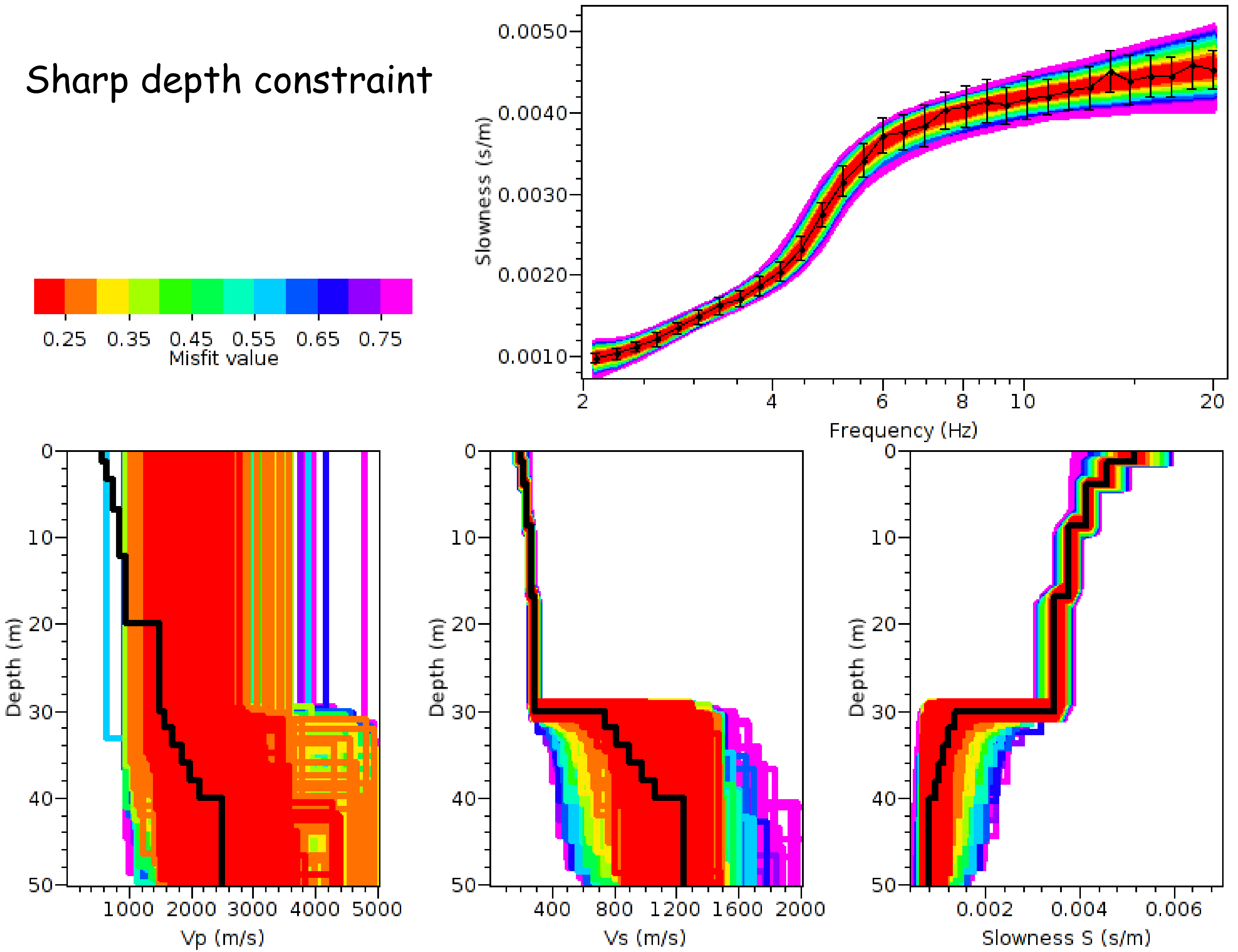
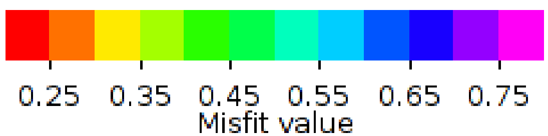
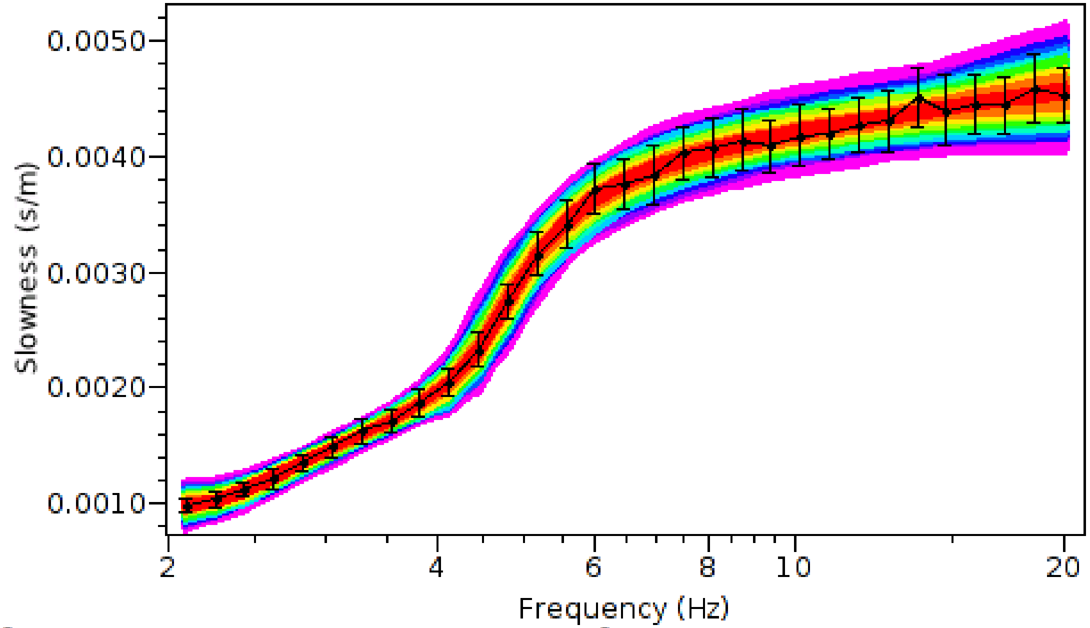


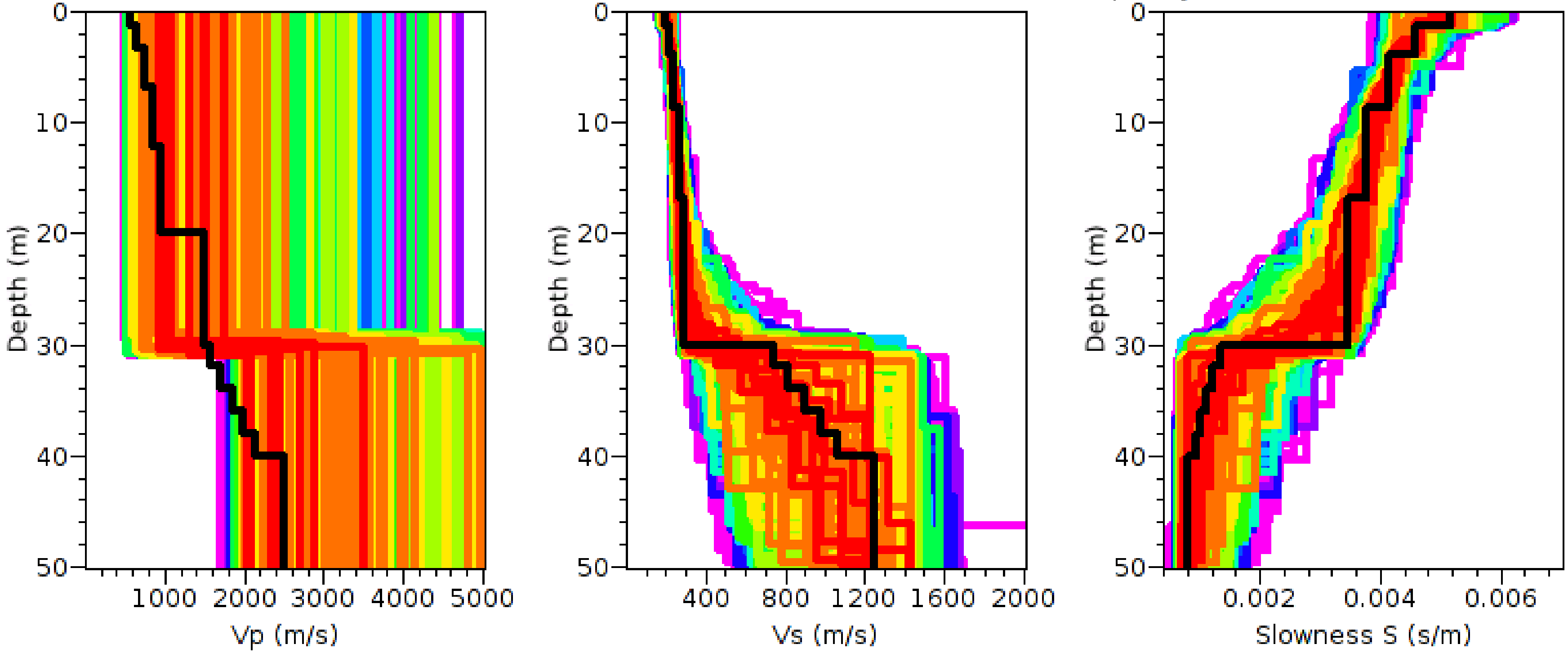
Sharp depth constraint



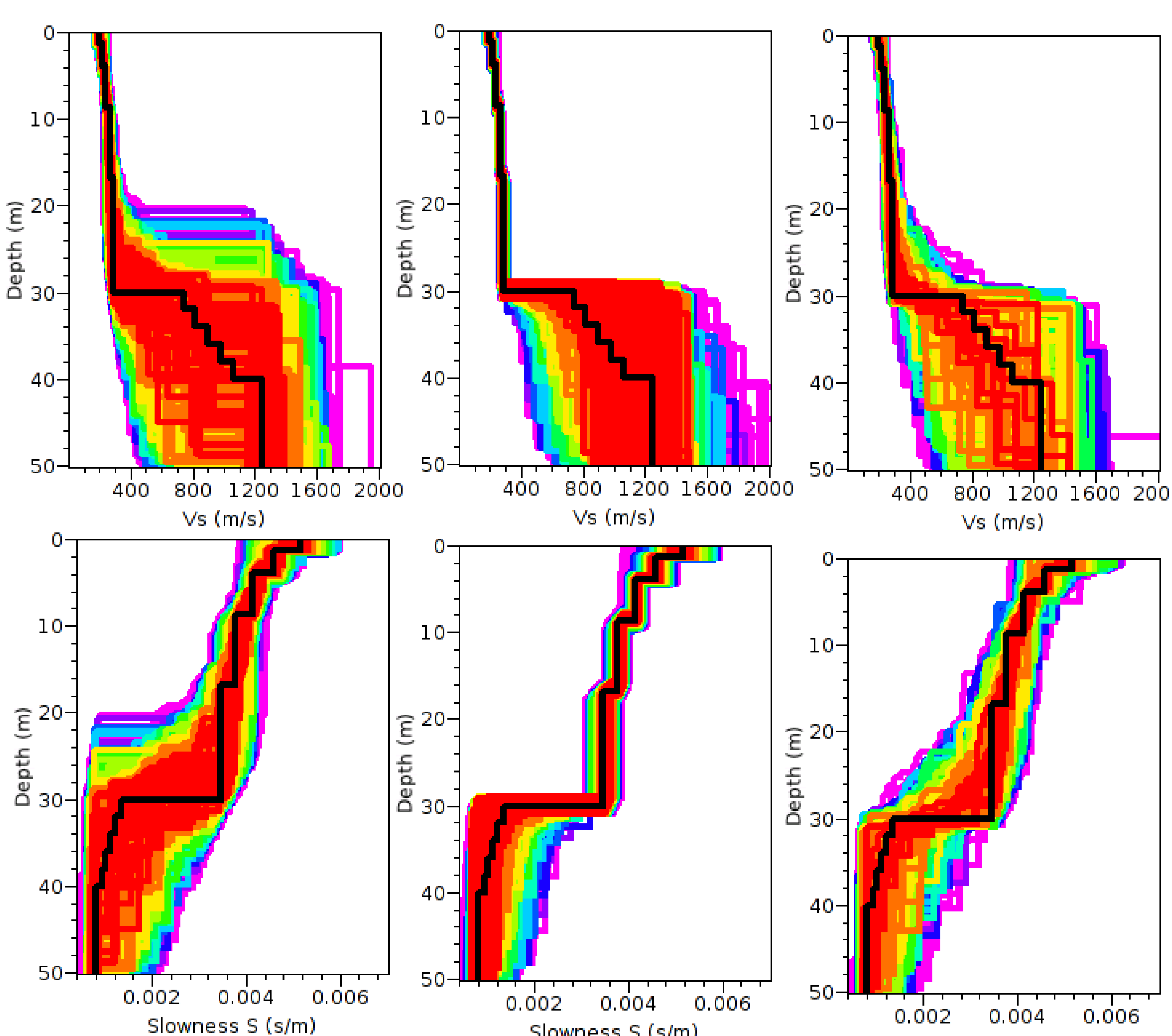
Smooth depth constraint





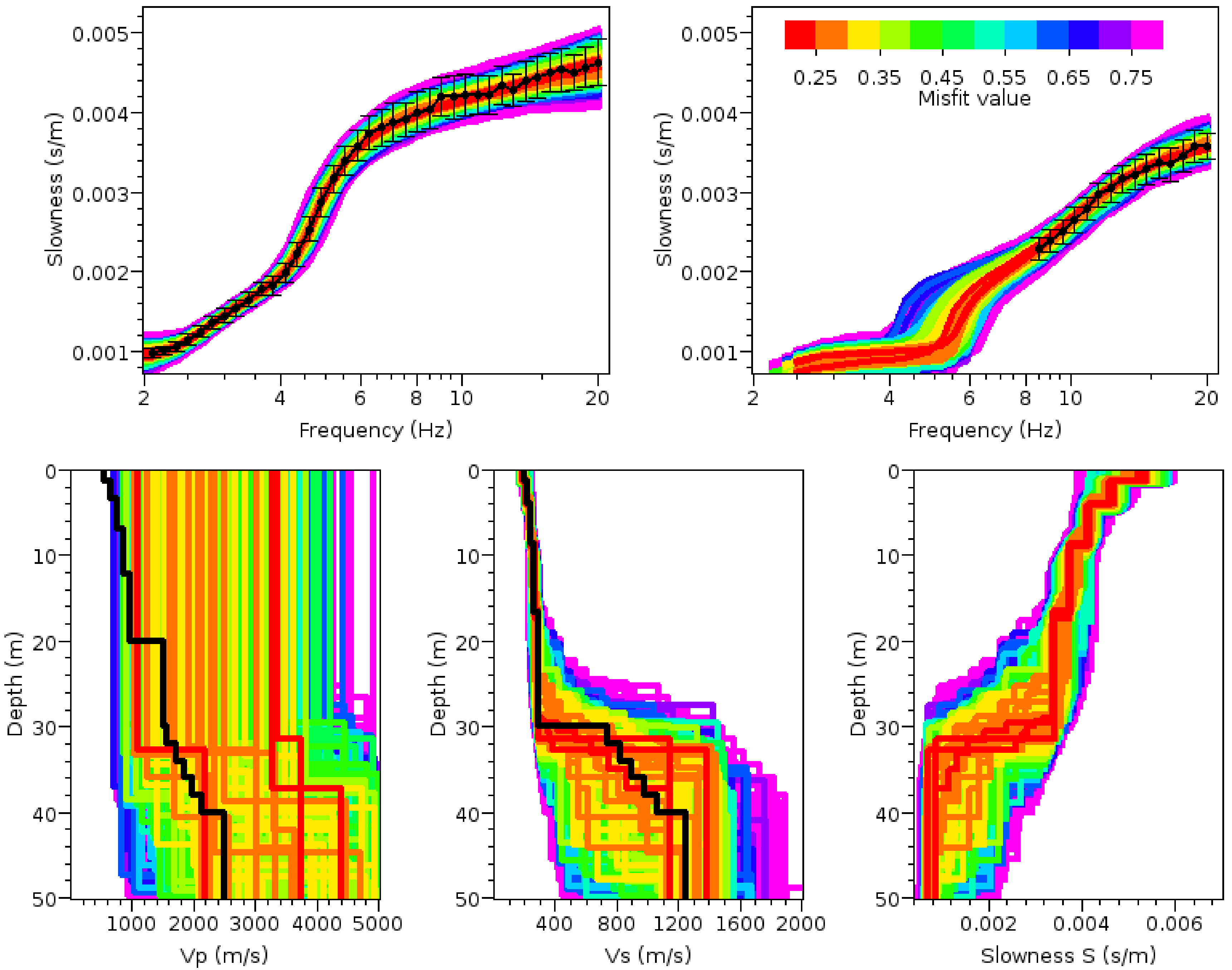


Effects of depth contraint



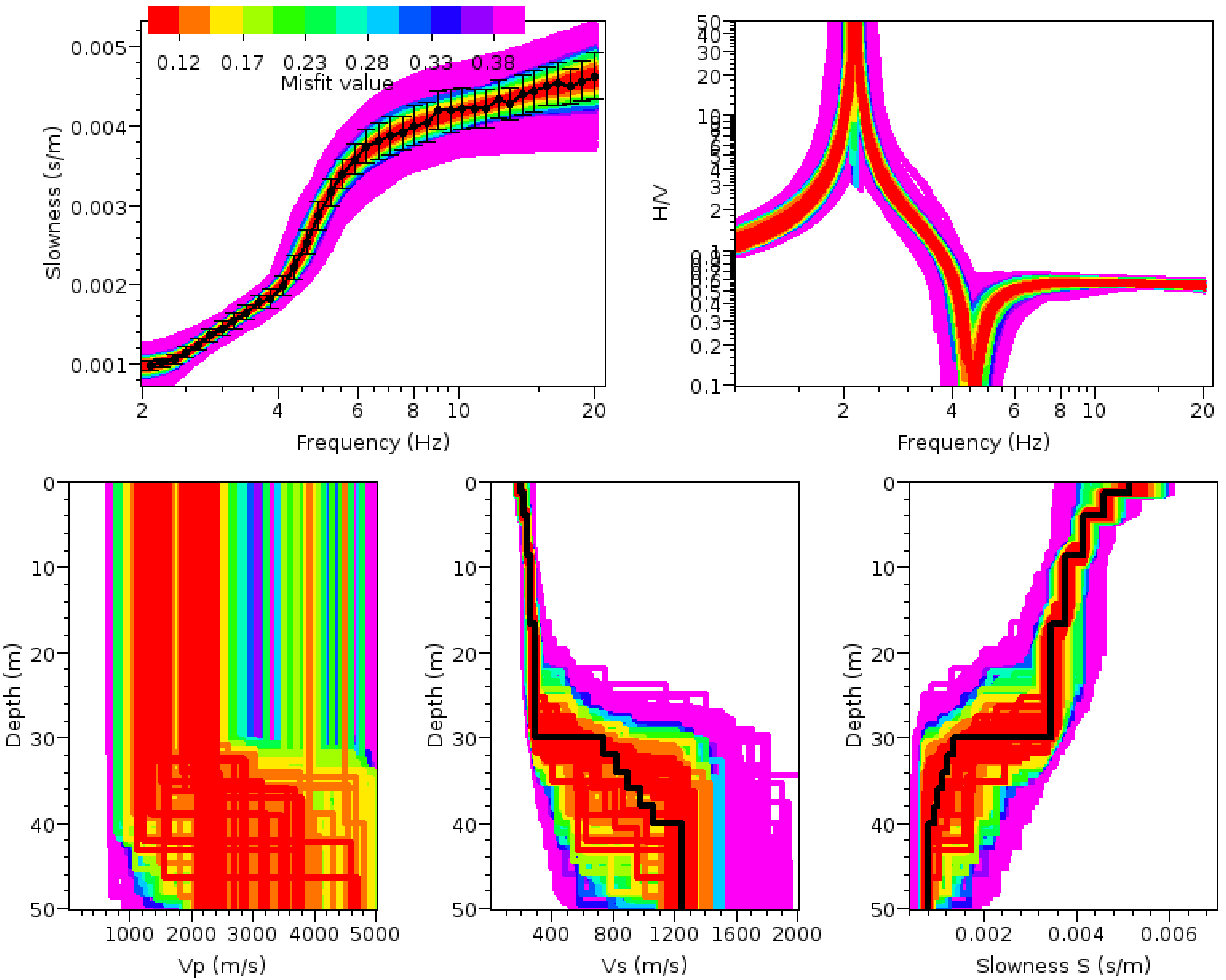
No constraint Sharp depth Smooth depth

Higher mode

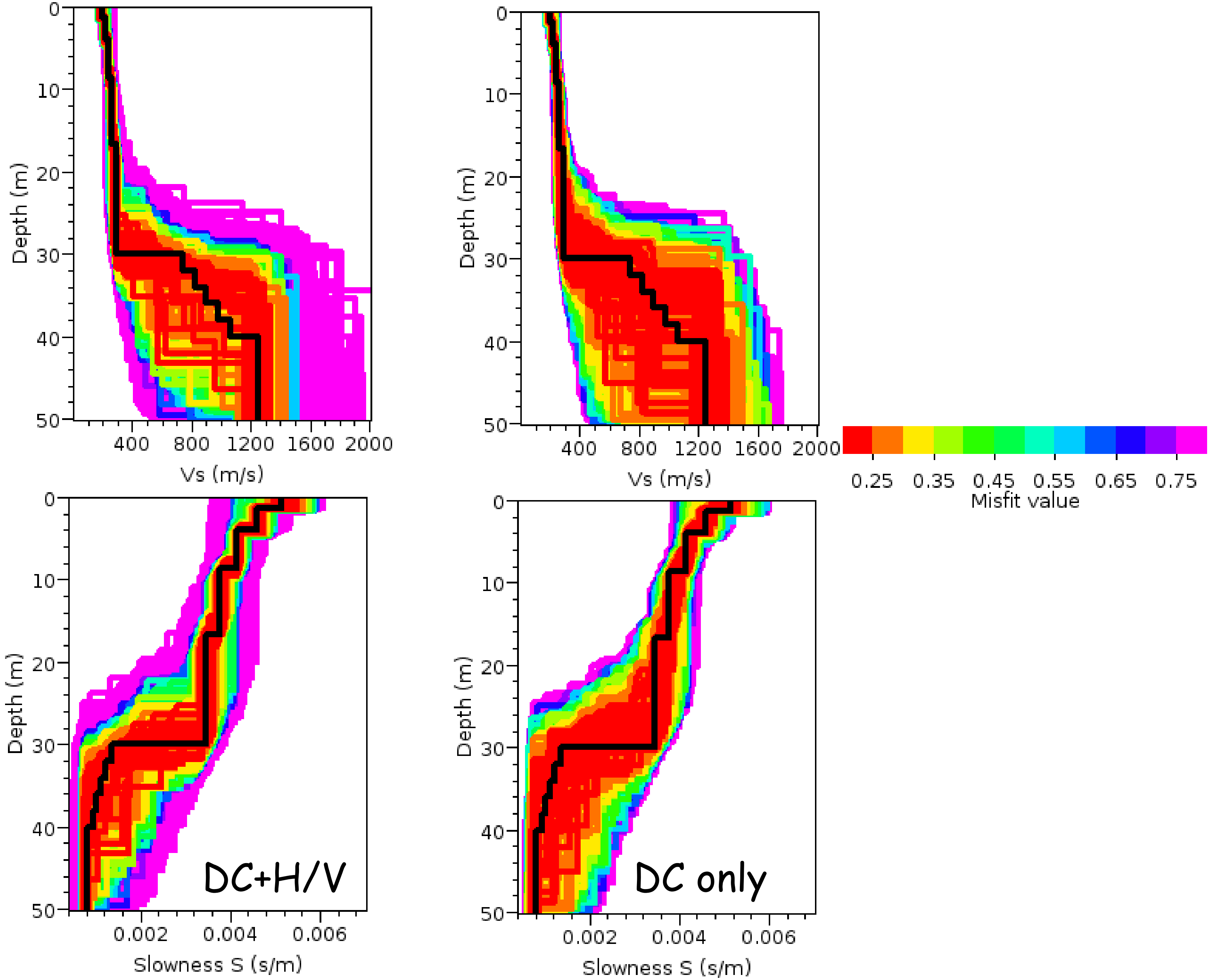


0.005
0.004
0.003
0.002
0.001
Slowness (s/m)
2
4
6
8
10
20
Frequency (Hz)
0.25
0.35
0.45
0.55
0.65
0.75
Misfit value
0
10
20
30
40
50
Depth (m)
1000
2000
3000
4000
5000
Vp (m/s)
400
800
1200
1600
2000
Vs (m/s)
0.002
0.004
0.006
Slowness S (s/m)

Joint inversion of H/V peak



Joint inversion of H/V peak



Conclusions

- **New Neighborhood Algorithm for parameter spaces with irregular boundaries**
- **Exploration capabilities improved**
- Better exploration means also better data fit
- Less forward computations needed to achieve the same data fit
- Robust results: all seeds return the same model distribution