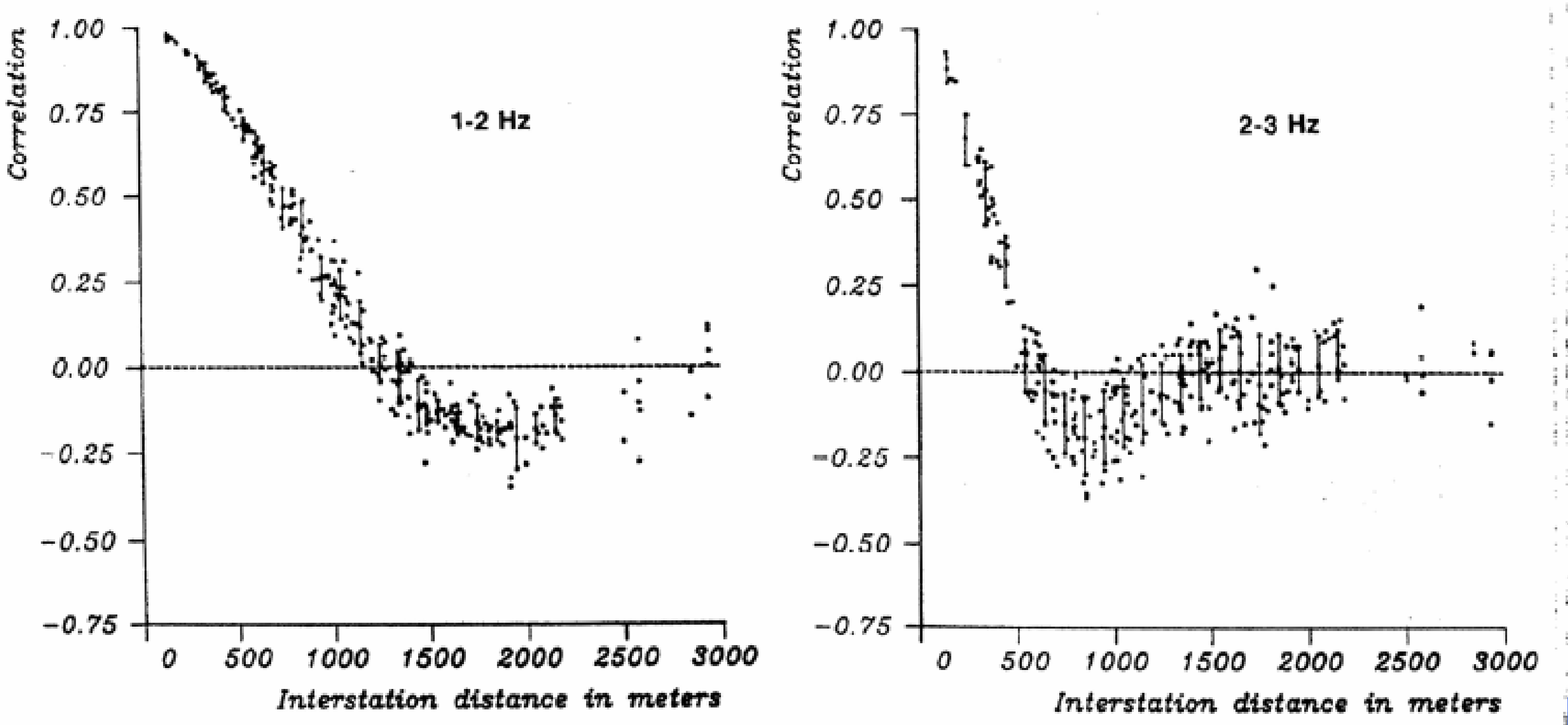
Using Ambient Vibration Array Techniques for Site Characterisation

Modified Autocorrelation method

(MSPAC)

NORES noise correlation analysis ⇨ coherence lengths



30 s long and taken at 05.15 h GMT on day 325 of 1985. Mean values and standard deviations within 100 m distance intervals are plotted on top of the population, except for short and long distances, where the number of correlation values is low.

Mykkeltveit, S., K. Åstebøl, D.J. Dornboos & E.S. Husebye (1983):
Seismic array configuration optimization. Bull. Seism. Soc. Am., 73: 173-186.

SPAC (Aki, 1957)

Hypothesis: Stochastic noise wavefield stationary in both time and space

Spatial correlation function

$$\phi(\mathbf{r},\varphi)=\frac{1}{\mathbf{T}}\int_0^{\mathbf{T}}\mathbf{u}(\mathbf{x},\mathbf{y},\mathbf{t})\mathbf{u}^*(\mathbf{x}+\mathbf{r}\cos\varphi,\mathbf{y}+\mathbf{r}\sin\varphi,\mathbf{t})\mathbf{dt}$$

Using relation between spectrum in time and spectrum in frequency, for one source with propagation azimuth θ

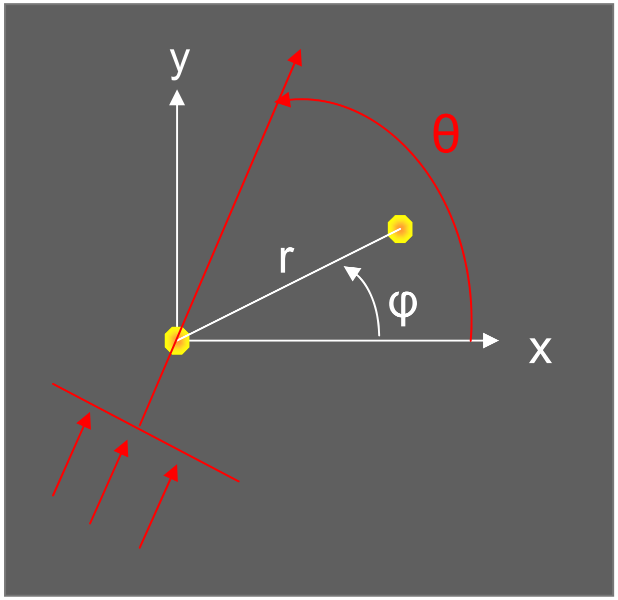
$$\phi(r,\varphi)=\frac{1}{\pi}\int_0^{\infty}\Phi(\omega)\cos\left[\frac{\omega r}{c(\omega)}\cos(\theta-\varphi)\right]d\omega$$

cross-spectrum

Narrow band filtering around w_o

$$\Phi(\omega)=\Phi(\omega_0)\delta(\omega-\omega_0)$$

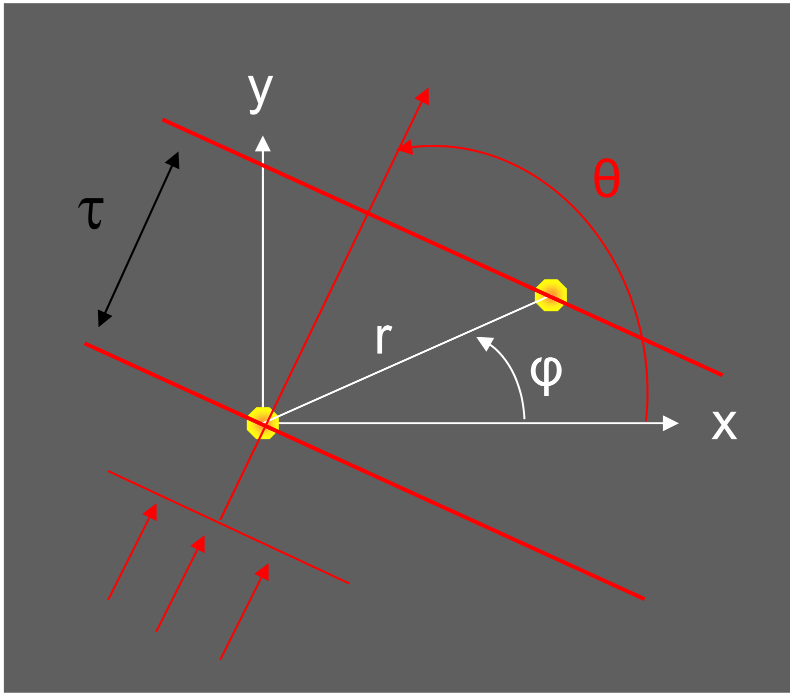
$$\phi(r,\varphi,\omega_0)=\frac{1}{\pi}\Phi(\omega_0)\cos\left[\frac{\omega_0 r}{c(\omega_0)}\cos(\theta-\varphi)\right]$$



Correlation coefficient

$$\rho(r,\varphi,\omega_0)=\frac{\phi(r,\varphi,\omega_0)}{\phi(0,\varphi,\omega_0)}=\cos\left[\frac{\omega_0\mathbf{r}}{\mathbf{c}(\omega_0)}\cos(\theta-\varphi)\right]$$

Correlation coefficient



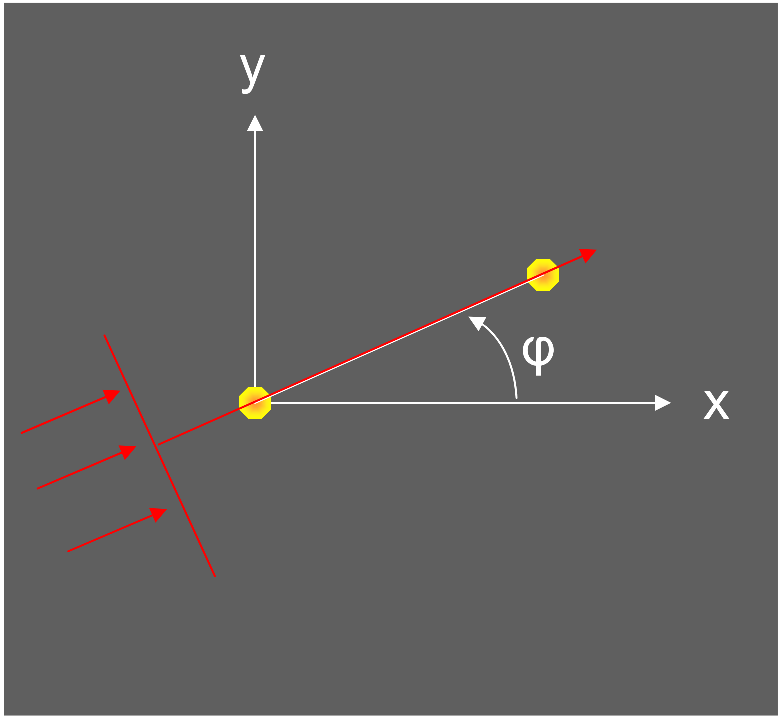
$$\rho(\mathbf{r},\varphi,\omega_0) = \cos\left[\frac{\omega_0 r}{c(\omega_0)}\cos(\theta-\varphi)\right]$$

$$\tau = \frac{\mathbf{r}}{\mathbf{c}(\omega_0)}\cos(\theta-\varphi)$$

Time delay

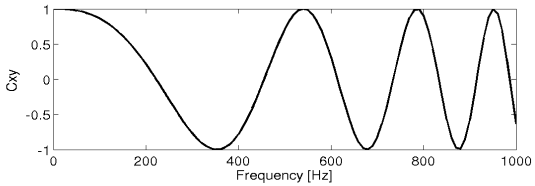
$$\rho(\mathbf{r},\varphi,\omega_0) = \cos[\omega_0\tau]$$

Correlation coefficient



$\theta - \varphi = 0$

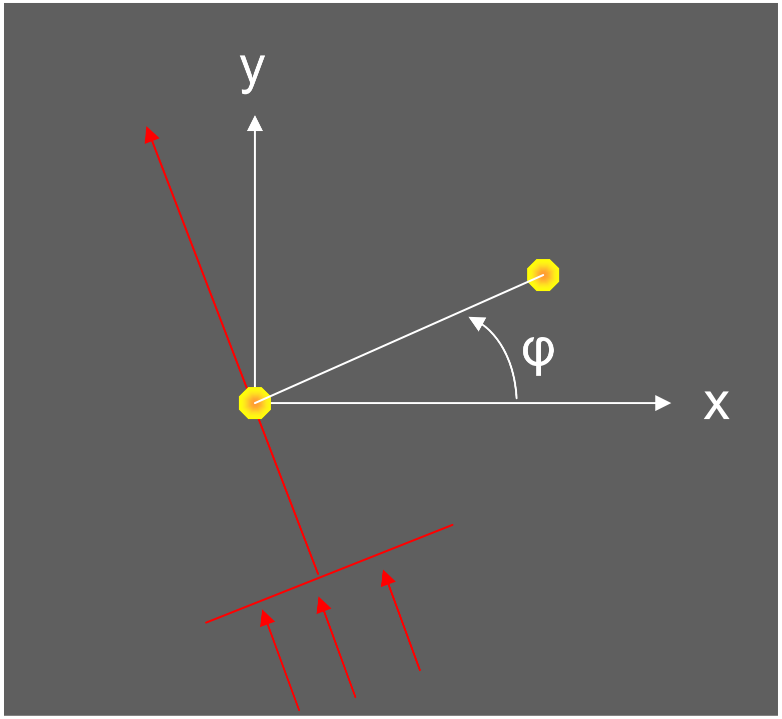
c(ω)



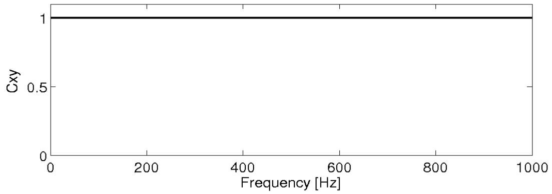
$$\phi(r,\varphi,\omega_0) = \cos\left[\frac{\omega_0 r}{c(\omega_0)}\cos(\theta-\varphi)\right]$$

Correlation coefficient

$c(\omega)$

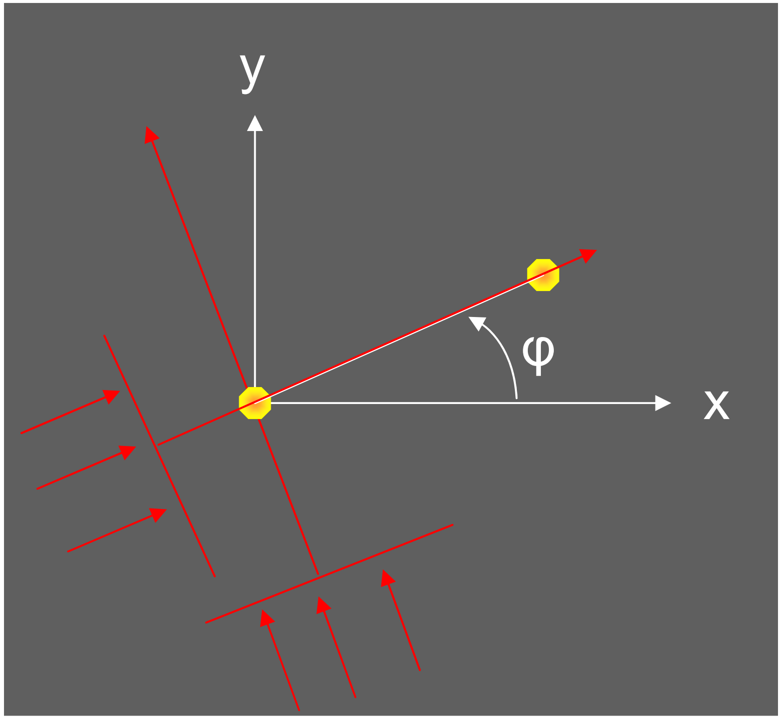


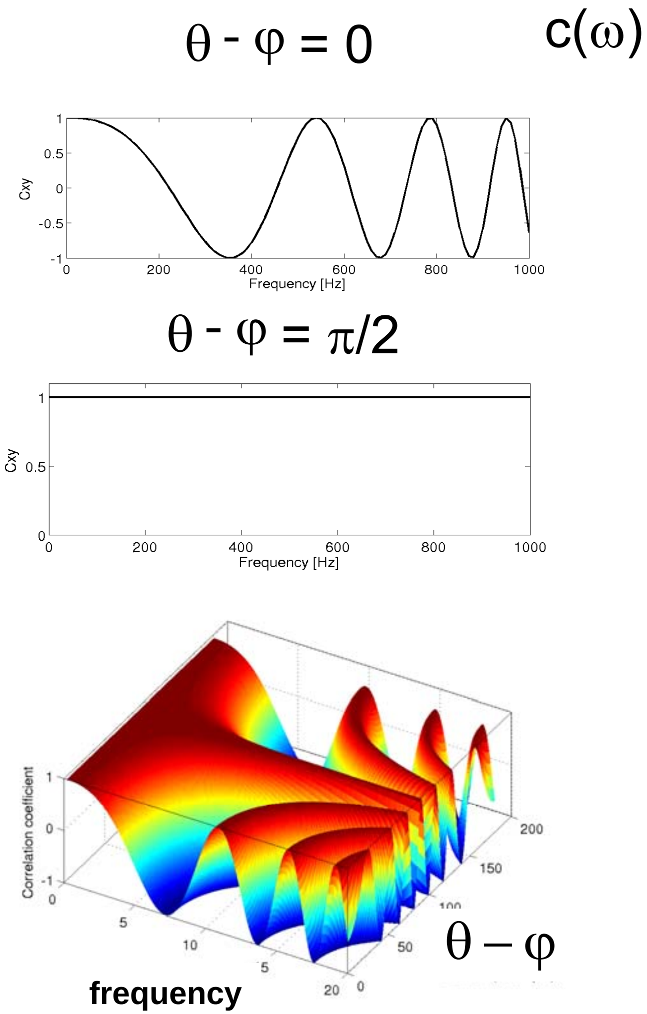
$\theta - \varphi = \pi/2$



$$\phi(r,\varphi,\omega_0) = \cos\left[\frac{\omega_0 r}{c(\omega_0)}\cos(\theta-\varphi)\right]$$

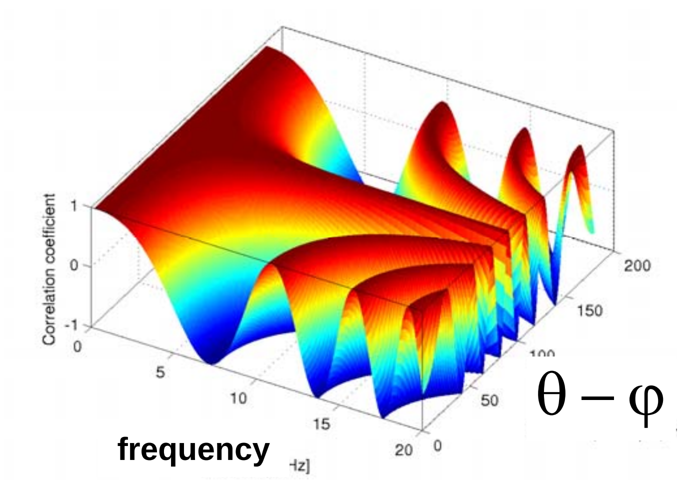
Correlation coefficient





$$\phi(r,\varphi,\omega_0) = \cos\left[\frac{\omega_0 r}{c(\omega_0)}\cos(\theta-\varphi)\right]$$

Correlation coefficient

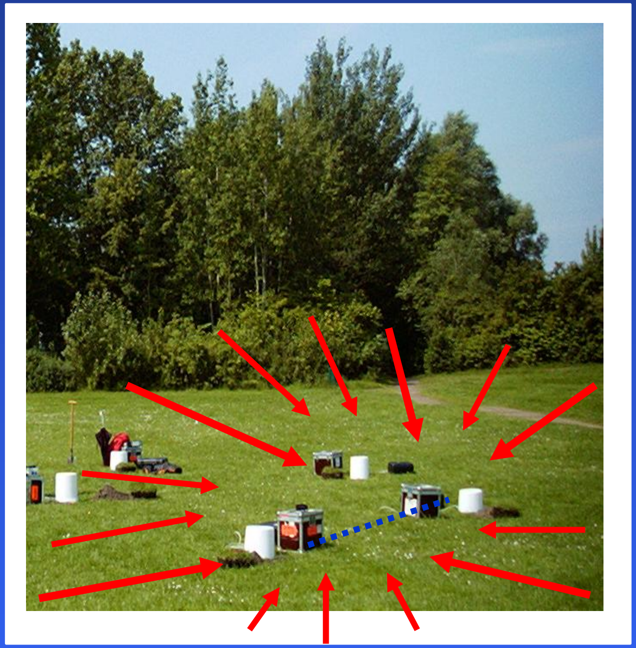


$$\phi(r,\varphi,\omega_0) = \cos\left[\frac{\omega_0 r}{c(\omega_0)}\cos(\theta-\varphi)\right]$$

For a single plane wave propagating with back-azimuth θ (except $\theta - \varphi = \pi/2$) and two sensors, $c(\omega_0)$ can be estimated

For ambient noise however, multiple sources (several θ) ….

Average spatial aucorrelation coefficient



For a large number of azimuthaly distributed sources

Azimuthal averaging of spatial autocorrelation coefficients

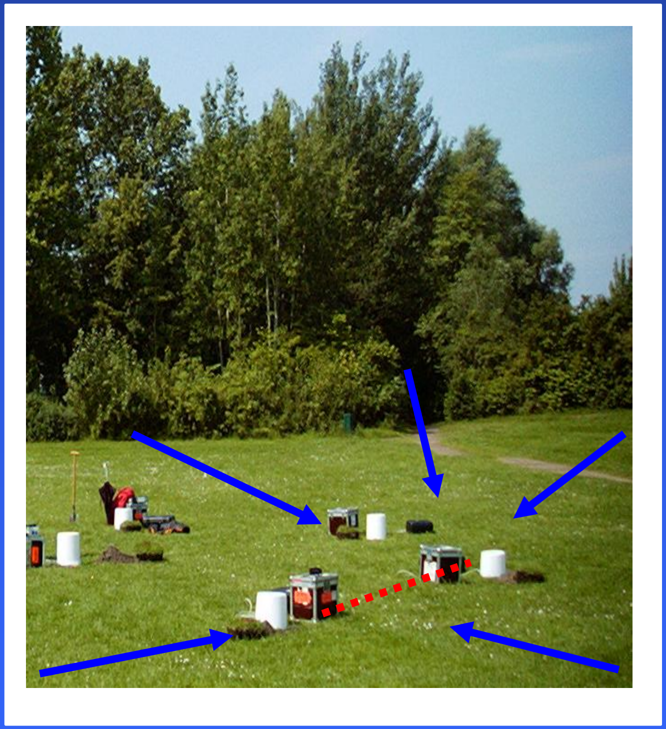
$$\bar{\rho}(\mathbf{r},\omega_0) = \frac{1}{\pi}\int_0^{\pi} \rho(\mathbf{r},\varphi,\omega_0)\mathbf{d}(\theta-\varphi)$$

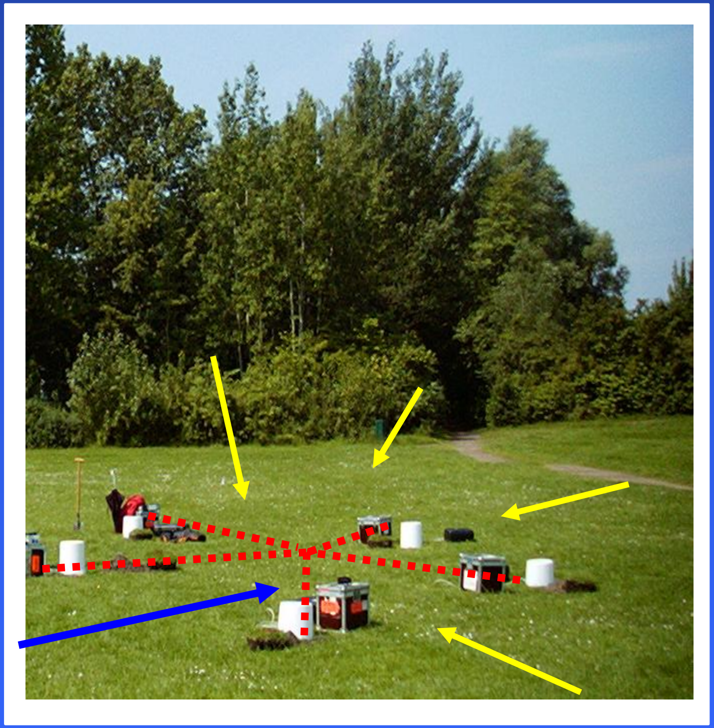
$$\bar{\rho}(r,\omega_0) = J_0\left(\frac{\omega_0 r}{c(\omega_0)}\right)$$

$$J_0(x) = 1/\pi \int_0^{\pi} \cos(x\cos(\varphi))\, d\varphi$$

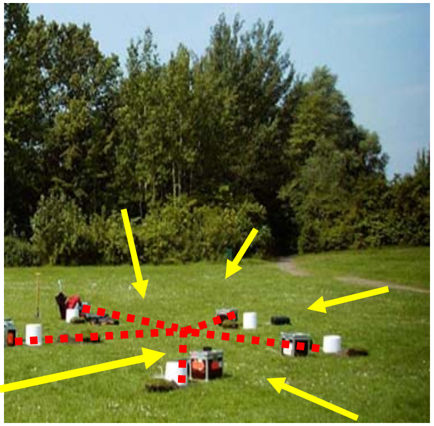
Average spatial aucorrelation coefficient (N-sensors case)

In ambient noise, number and spatial distribution of sources are unknowns …





Average spatial aucorrelation coefficient (N-sensors, circular arrays)



Azimuthal averaging of spatial autocorrelation coefficients

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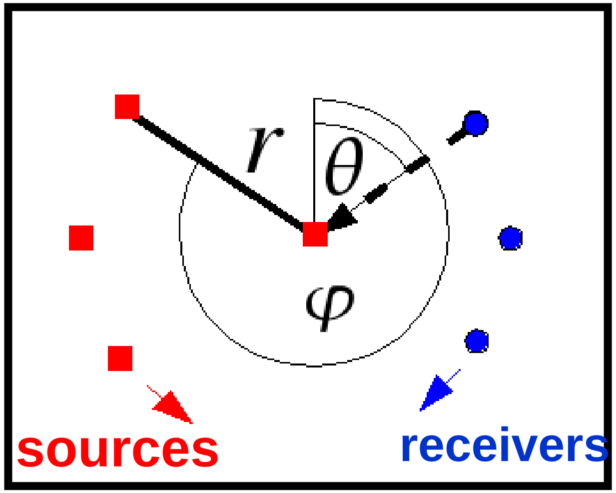
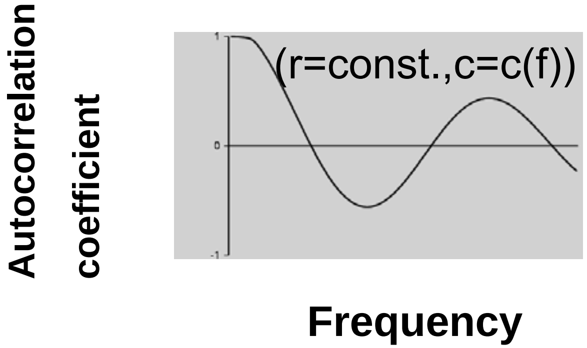
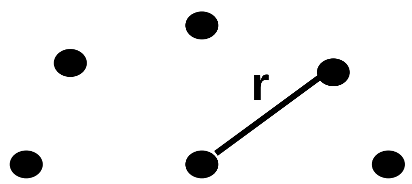


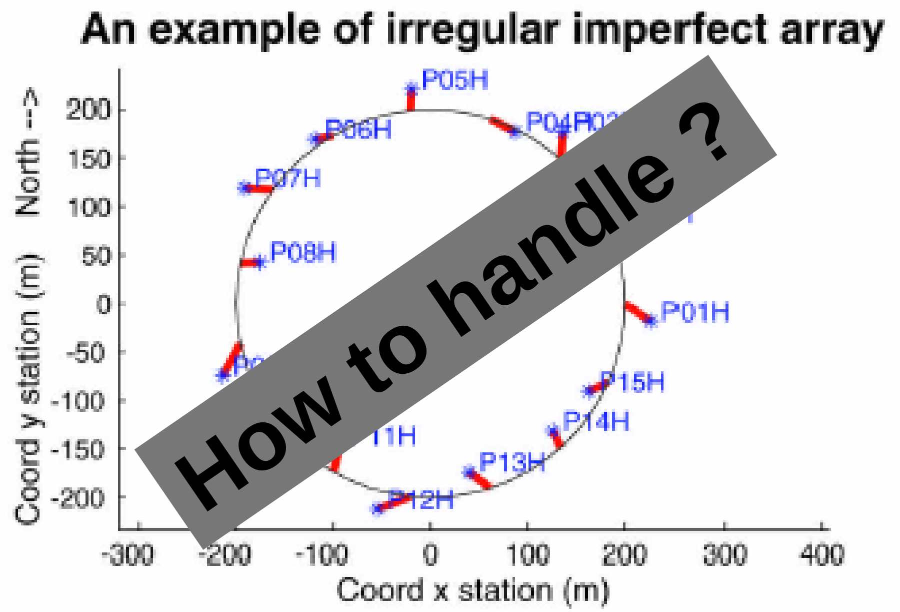


MSPAC (Bettig et al., 2001): extension to arbitary array configurations

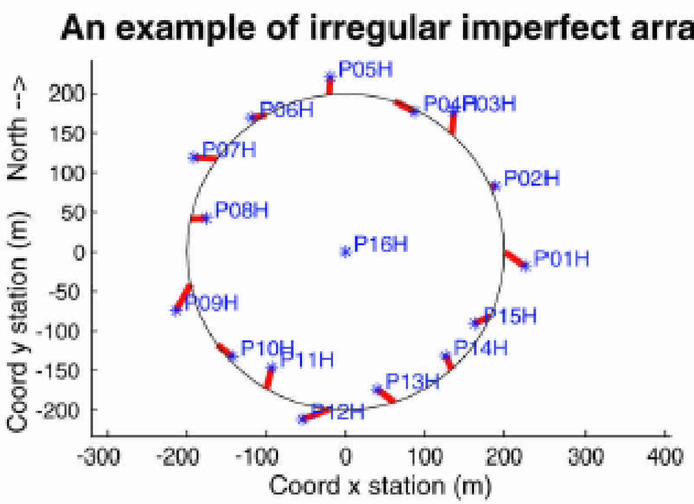
SPAC (Aki, 1957)



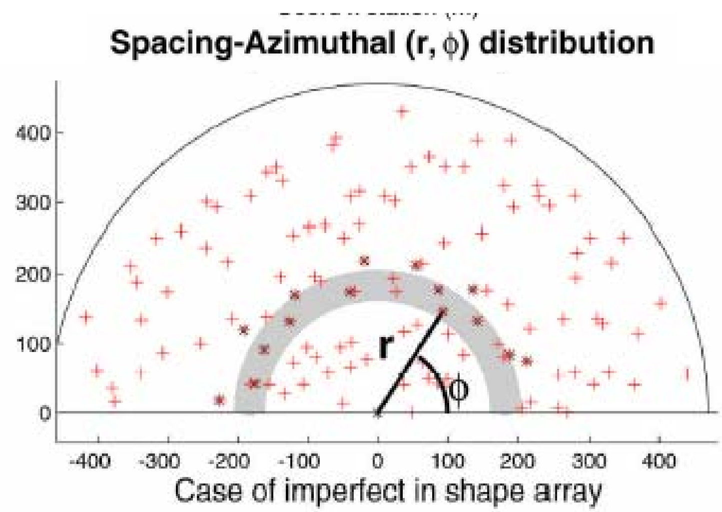
$$\overline{\rho}(r,\omega_0) = J_0\left(\frac{\omega_0 r}{c(\omega_0)}\right)$$



MSPAC (Bettig et al., 2001): extension to arbitary array configurations



co-arrays

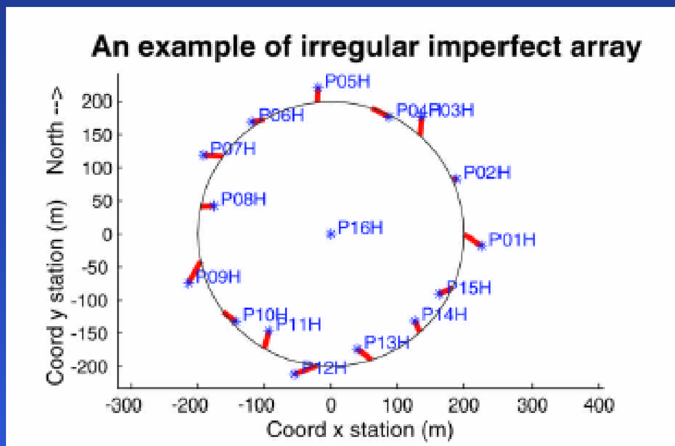


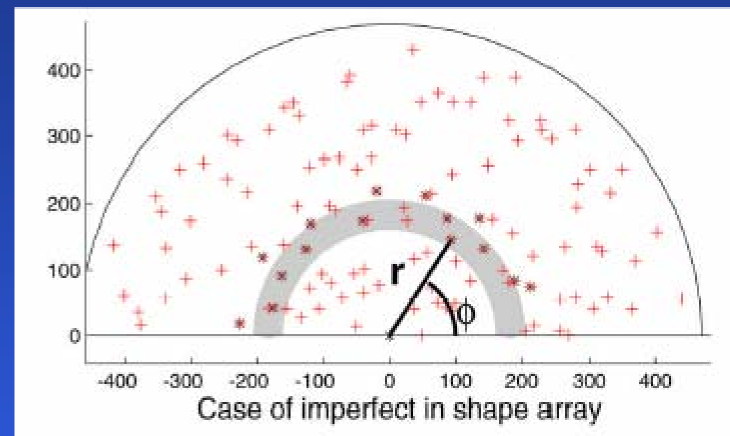
Computation of azimuthal and **radial** averaged spatial autocorrelation coefficients

$$\overline{\rho_{z,r1,r2}}(\omega_0)=\frac{2}{r_2^2-r_1^2}\cdot\frac{c_R(\omega_0)}{\omega_0}\cdot\left[r\cdot J_1\left(\frac{\omega_0\cdot r}{c_R(\omega_0)}\right)\right]_{r_1}^{r_2}$$

With r_1 and r_2 being the inner and outer radius of the ring, respectively

MSPAC vs. SPAC: advantages and drawbacks





Advantages

- in urban areas: easier arrays realisation
- analysis of data sets suitable/optimized for FK
- broader frequency band than SPAC (only 1 ring)

Disavantages

- less « precise » estimation than SPAC

Estimation of phase velocity

$$\overline{\rho}(r,\omega) = J_0\left(\frac{\omega r}{c(\omega)}\right)$$

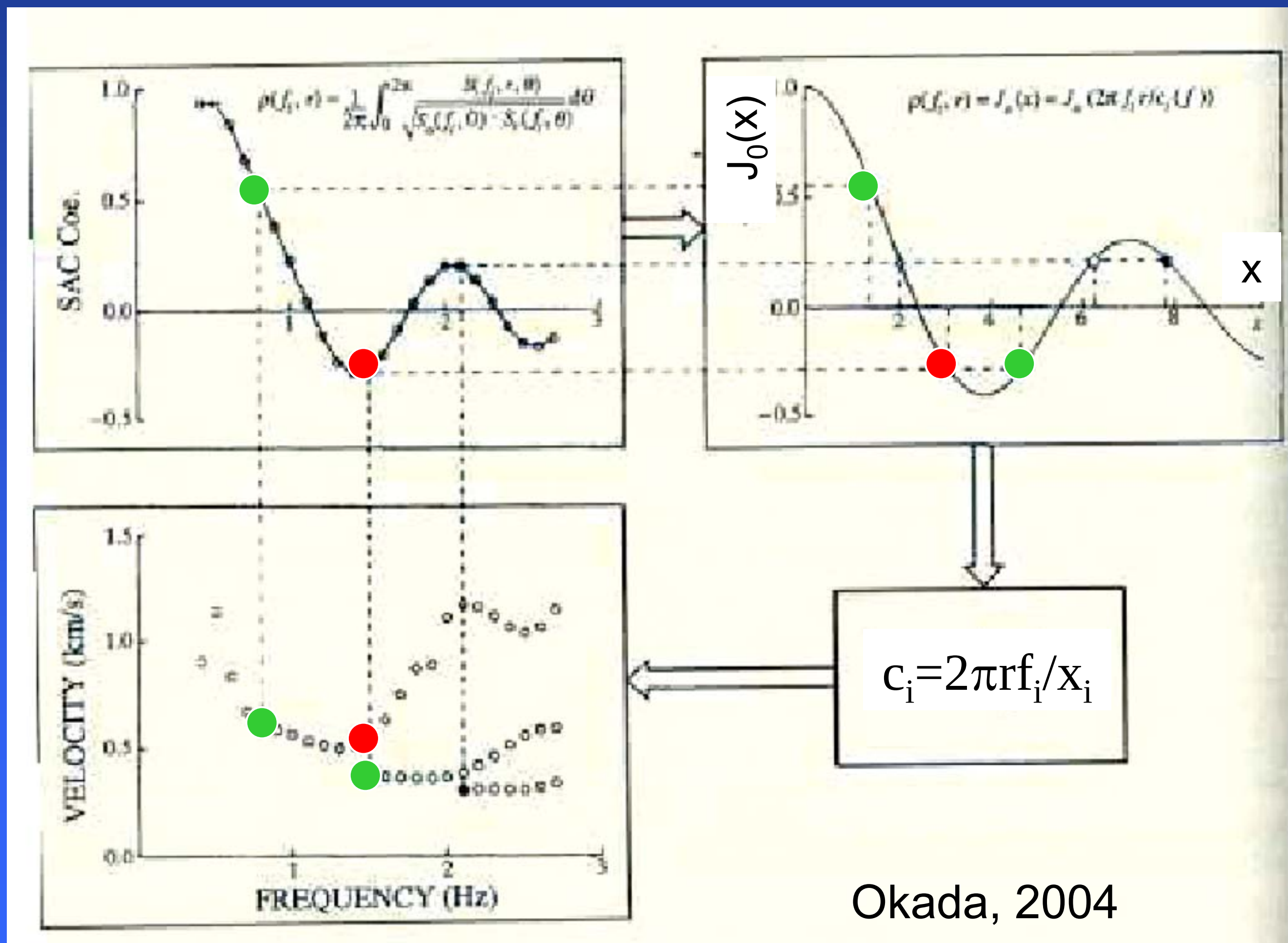
measure

target

Correlation coeficient

Phase velocity

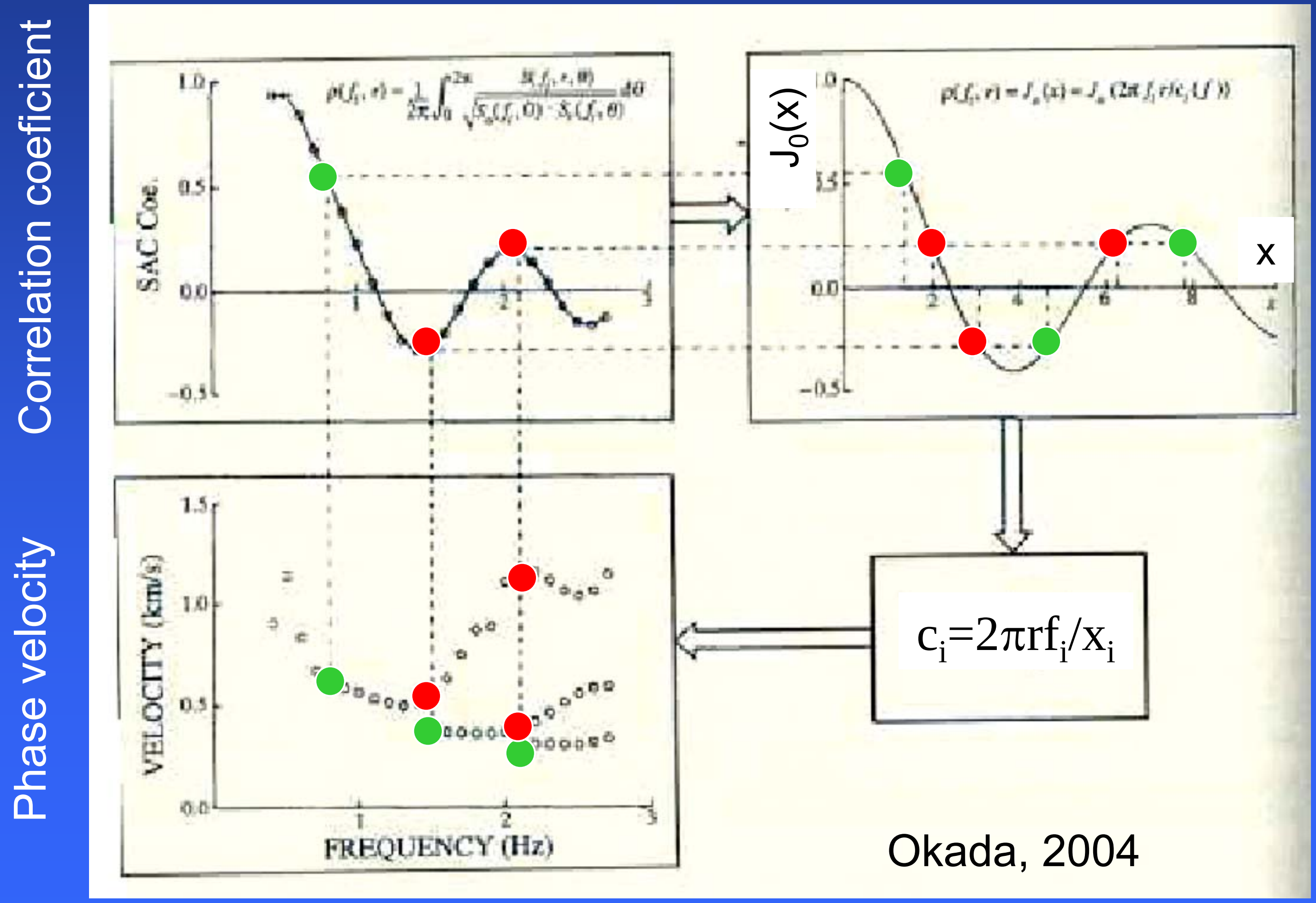
Bessel function



Estimation of phase velocity

$$\overline{\rho}(r,\omega) = J_0\left(\frac{\omega r}{c(\omega)}\right)$$

measure target

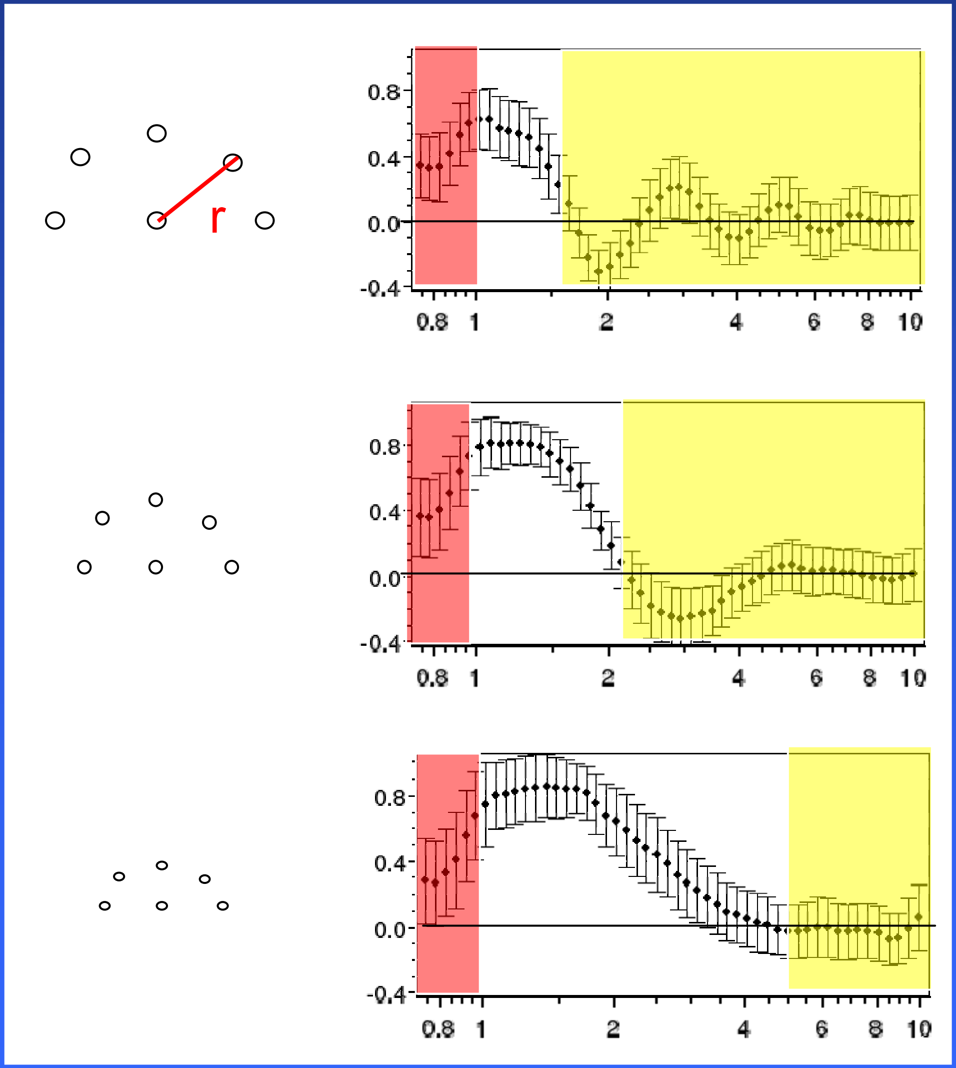


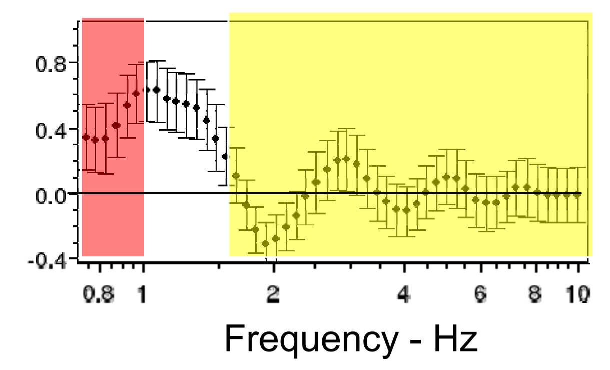
Building a broad-band dispersion curve from SPAC:

Resolution and non-uniqueness issues

December 6th-12th 2008, Thessaloniki, Greece

Spatial autocorrelation coefficient for different radius : resolution and non-uniqueness

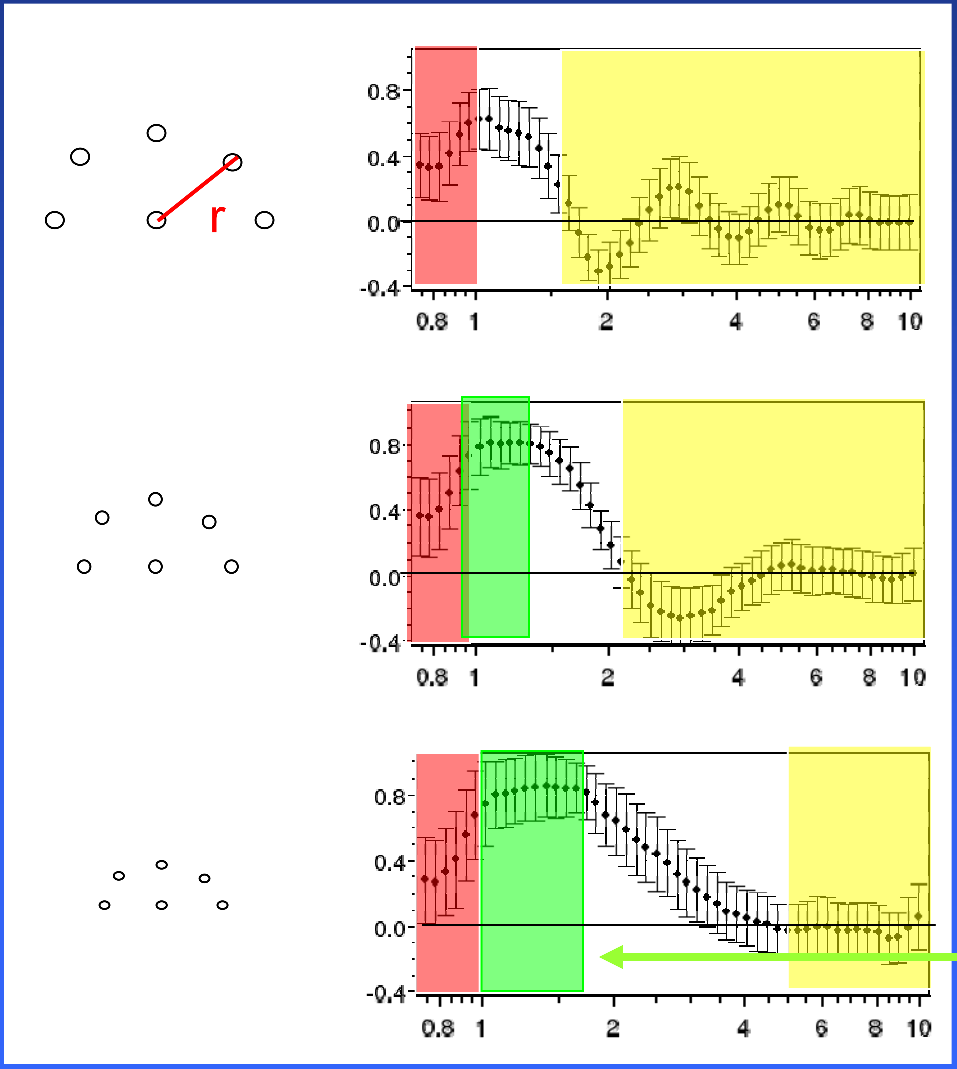


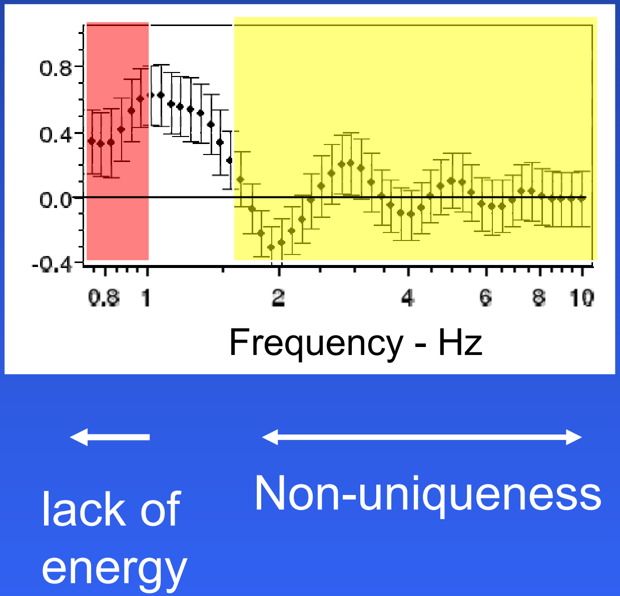


lack of energy

Non-uniqueness

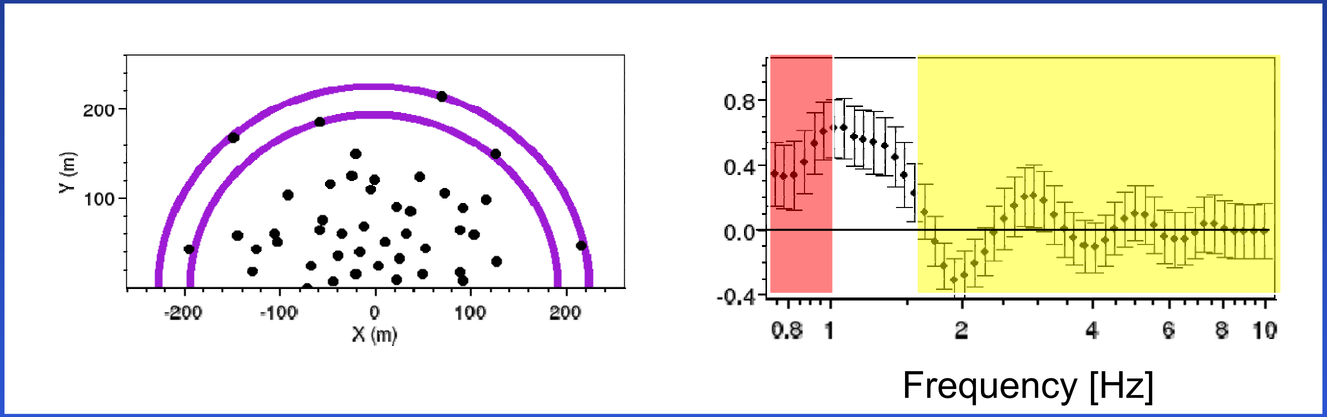
Spatial autocorrelation coefficient for different radius : resolution and non-uniqueness





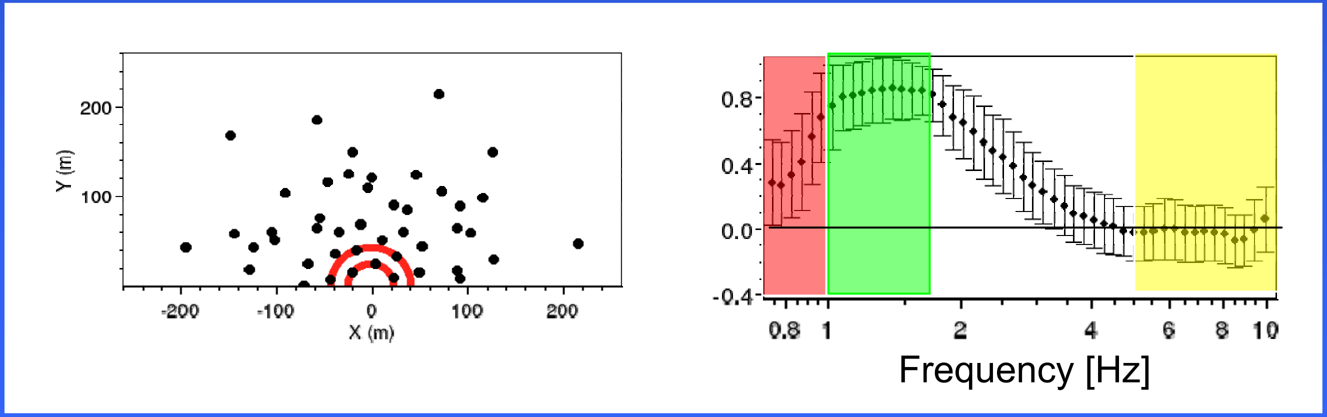
Lack of resolution

Array geometry: resolution and non-uniqueness



Largest ring controls the resolution

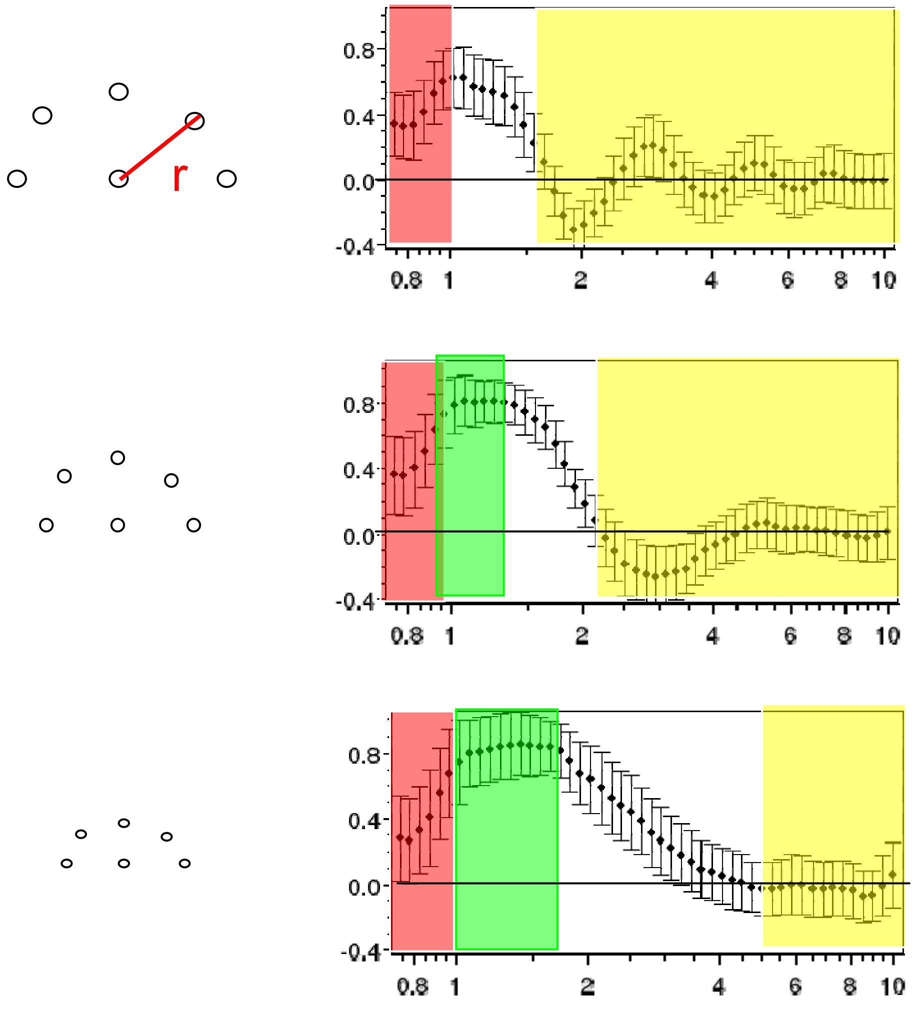
⇔ width of the f-k transfer function lobe

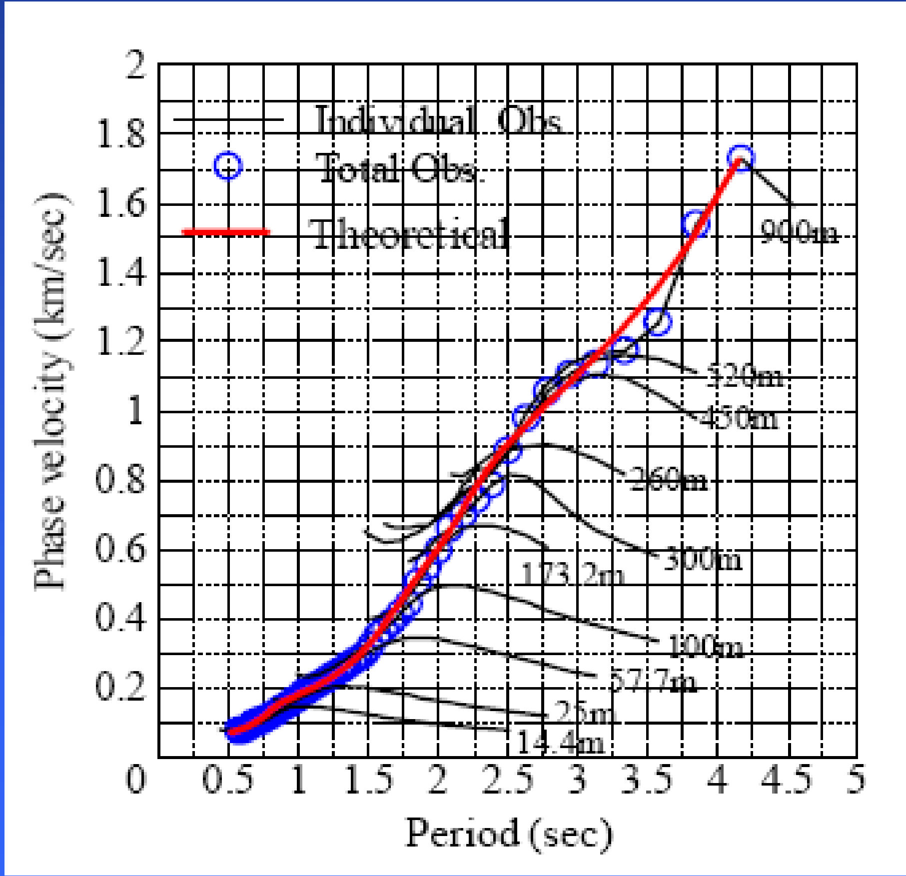


Smallest ring controls the non-uniqueness limit

⇔ location of f-k side lobes

Spatial autocorrelation coefficient for different radius : building a dispersion curve





Tsuno and Kudo, 2004

Trade-off between array aperture and smallest distance

-small distances/rings -> less non-uniqueness (first minimum at higher frequencies)

- large aperture/rings -> better resolution (plateau of Bessel functions)

Trade-off between thin rings and good station pair distribution

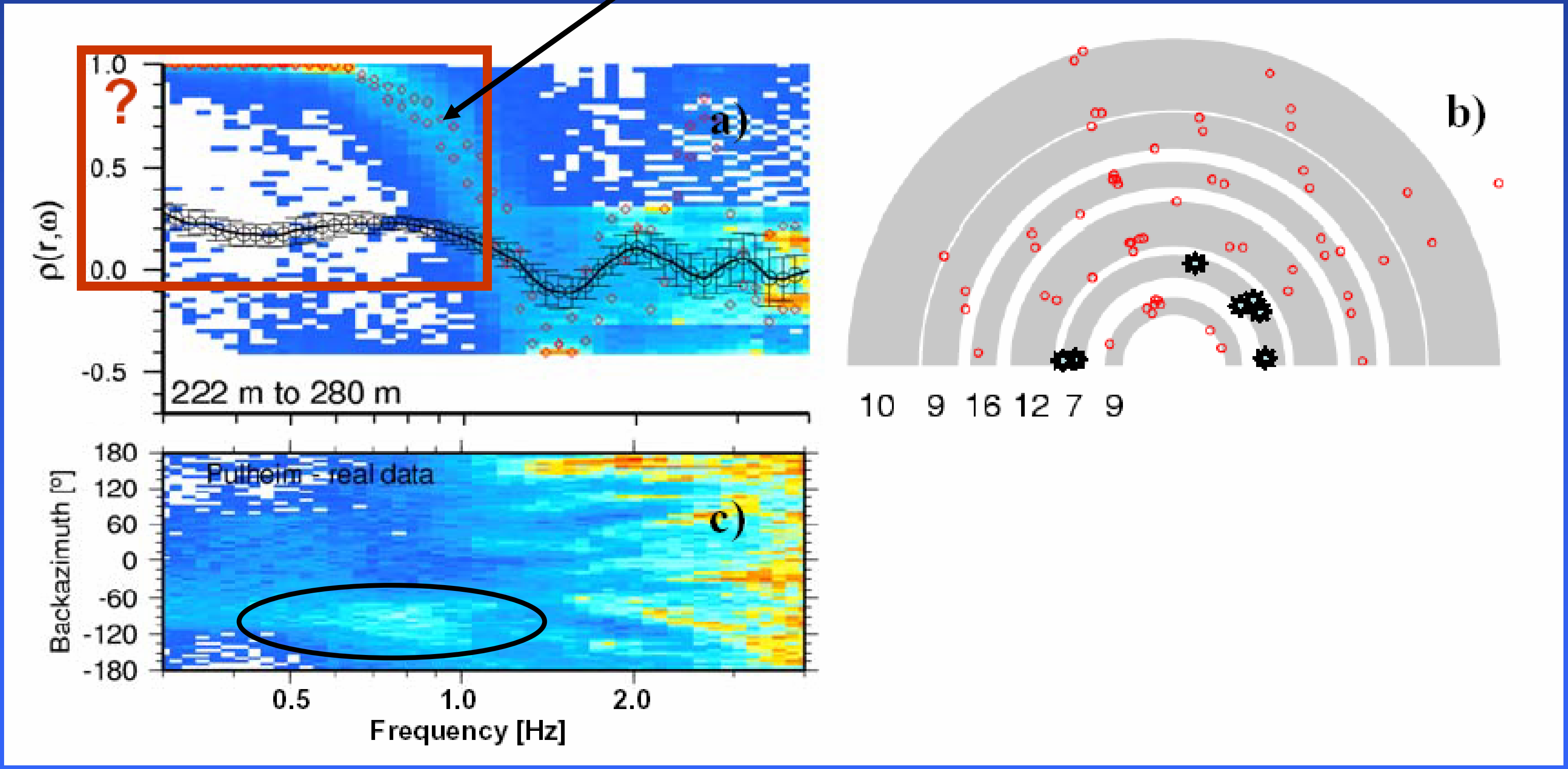
-Determination of inner and outer radius results from a compromise between the number of station pairs per ring (azimuthal resolution) and the ratio between ring thickness and ring radius

Finally, what are the main differences between FK and SPAC approaches ?

FK	SPAC
- plane wave propagation	- plane wave propagation
- one single source (or few uncorrelated sources)	- large number of uncorrelated sources spatially and temporally randomly distributed
- direct measure of phase delay	- measure of spatial (de-)coherency of waveforms

Effects of preferential sources direction on the average spatial autocorrelation coefficient

FK analysis



Ohrnberger (2004)

How to compute the autocorrelation coefficient?

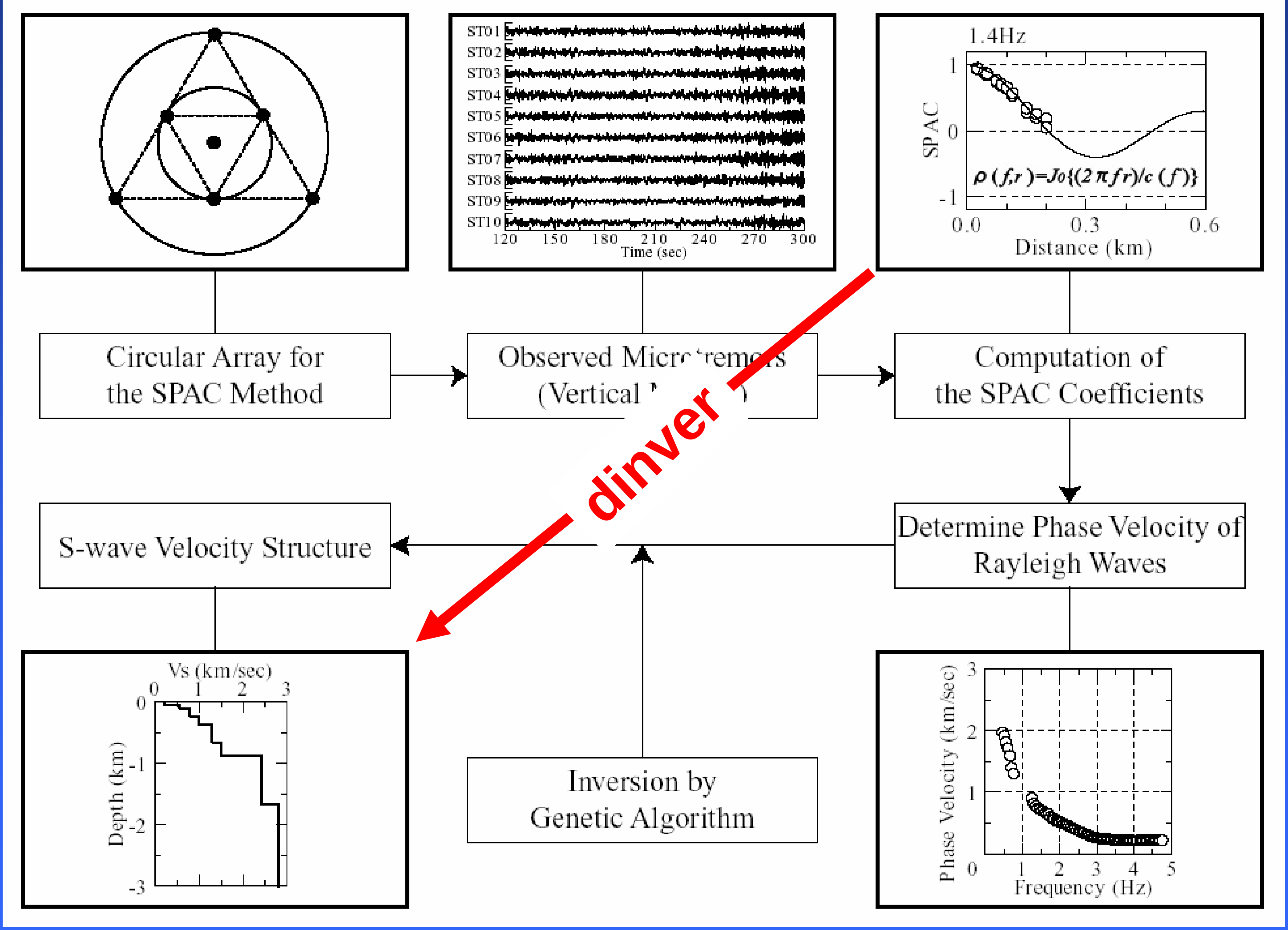
In the time domain (<=> sesarray)

- Compute the Fourier transform of signals
- Narrow band-pass filter around ω_0
- Inverse Fourier transform and computation of correlation
- Azimuthal averaging of the correlation coefficient

In the frequency domain

- Compute the cross-spectra normalized by the autospectra
- Azimuthal averaging on the real part of the cross-spectra

SPAC implementation



Kudo et al., 2002