ENGINEERING SEISMOLOGY & HAZARD ANALYSIS 2019

TUTORIAL 1

SIGNAL PROCESSING

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What Signal? & Why?

- Sound waves → High frequency, hearable range (20 Hz – 20,000 Hz)
- Electricity → (50 Hz – 60 Hz)
- Seismic Signals → (0.07 Hz to 50 Hz)

→***Question #1:*** How would you measure the damage on a structure after a strong ground motion?

→***Question #2:*** What does <u>AvgSa(T)</u> mean in the time domain physically?

Fourier Analysis

Time signals can be decomposed in to series of orthogonal cosine and sine functions with finite amplitudes and frequencies. The Fourier Analysis is the mathematical operation done for crossing between time and frequency domains. By taking the Fourier transform, a time signal can be transferred in to frequency space, interpreted, processed and reverted back to time domain without information loss.

$$F(v)=\int_{-\infty}^{\infty} f(t)\exp(-i2\pi vt)\,dt$$ Forward Fourier Transform

$$f(t)=\int_{-\infty}^{\infty} F(v)\exp(i2\pi vt)\,dt$$ Inverse Fourier Transform

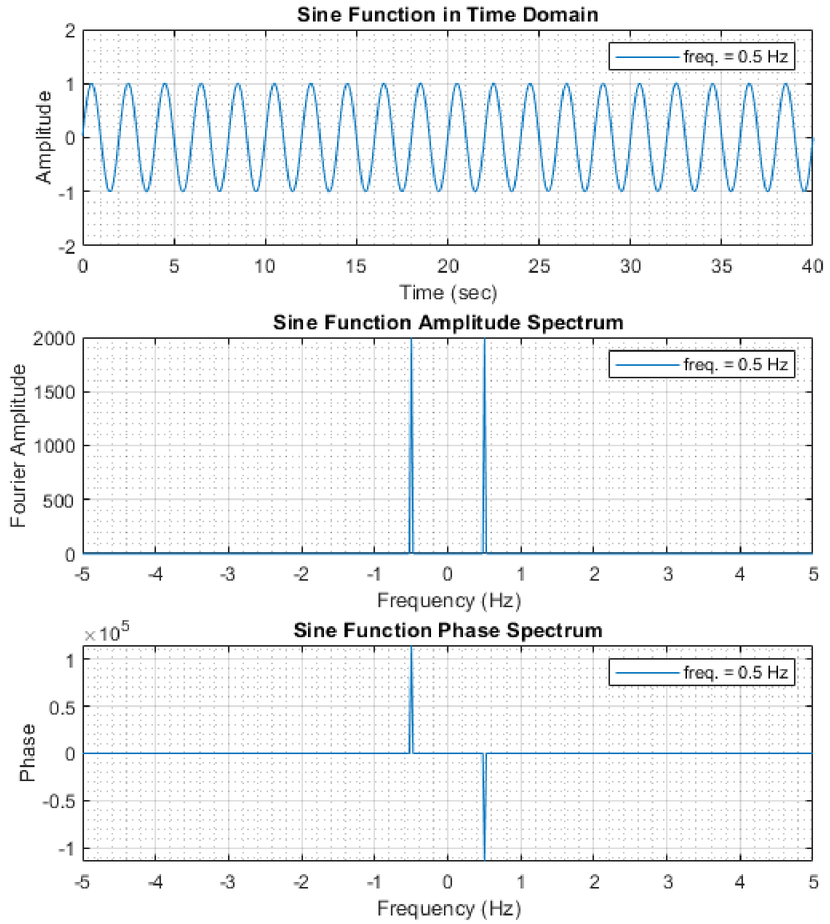
Recall! → Euler's Identity: $\mathrm{e}^{\mathrm{i}2\pi} = \cos\pi + \mathrm{i}\,\sin\pi$

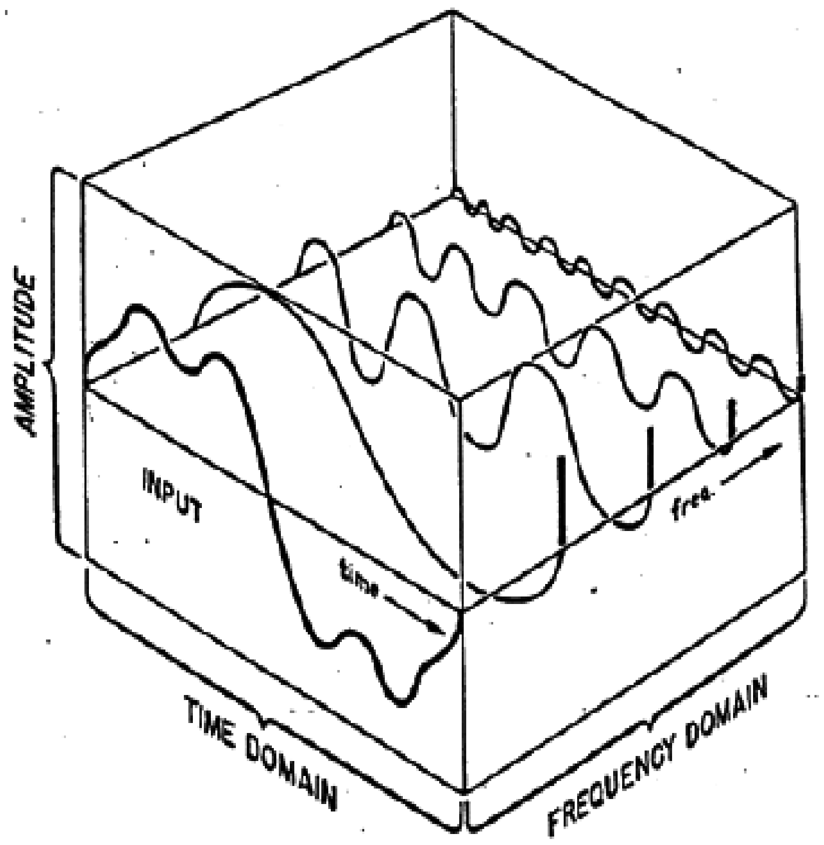
Fourier Analysis

Important Properties of the Fourier Transform:

- Linearity: $$F(\lambda f+\mu g)=\lambda F(f)+\mu F(g)=\lambda F(\omega)+\mu F(\omega)$$
- Integration: $$F[f'(t)]=i\omega.F(\omega) \rightarrow F[f^{(n)}(t)]=(i\omega)^n.F(\omega)$$
- Convolution: $$F[f*g]=F[\int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau]=F(\omega).G(\omega)$$
- Time Reversal: $$F[f(-t)]=F(-\omega)$$

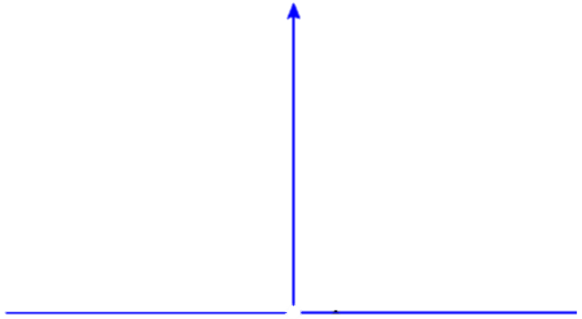
Frequency Domain



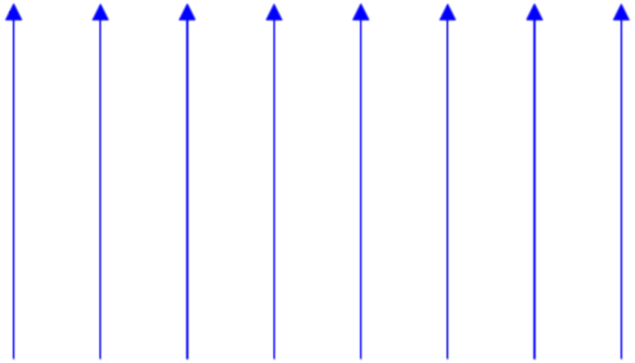


Useful Functions

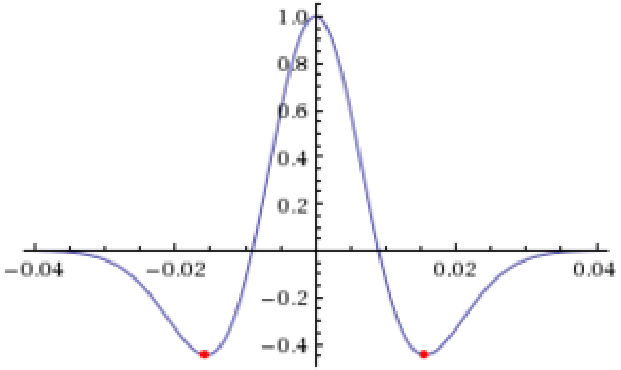
Boxcar



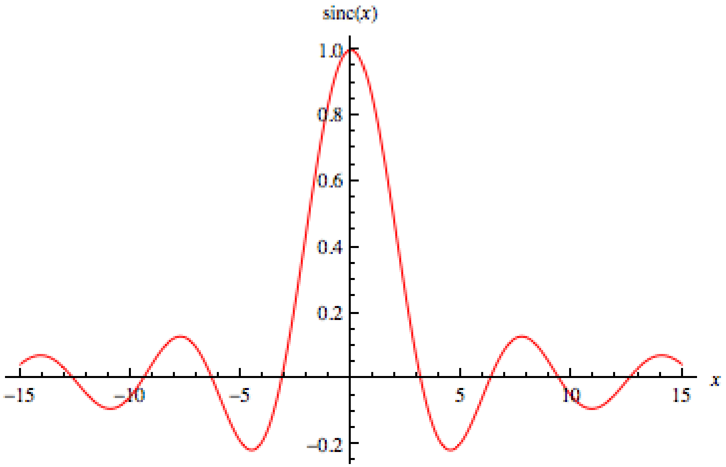
Dirac (Delta)



Dirac Comb



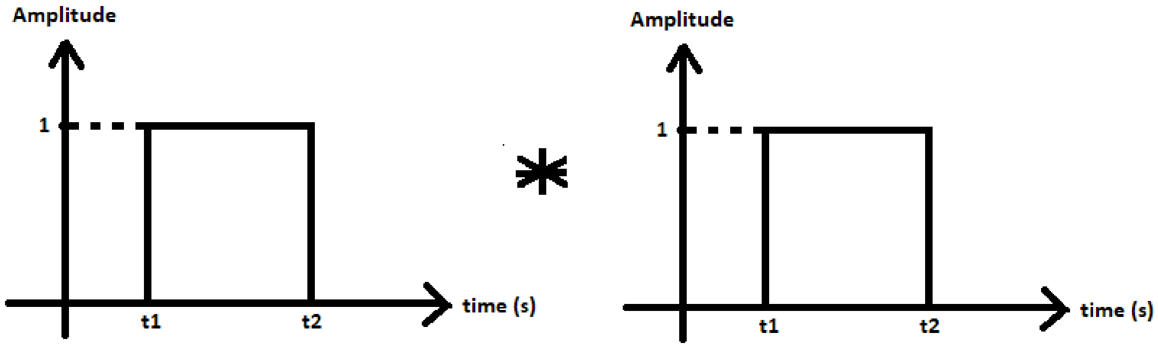
Mexican Hat (Ricker Wavelet)



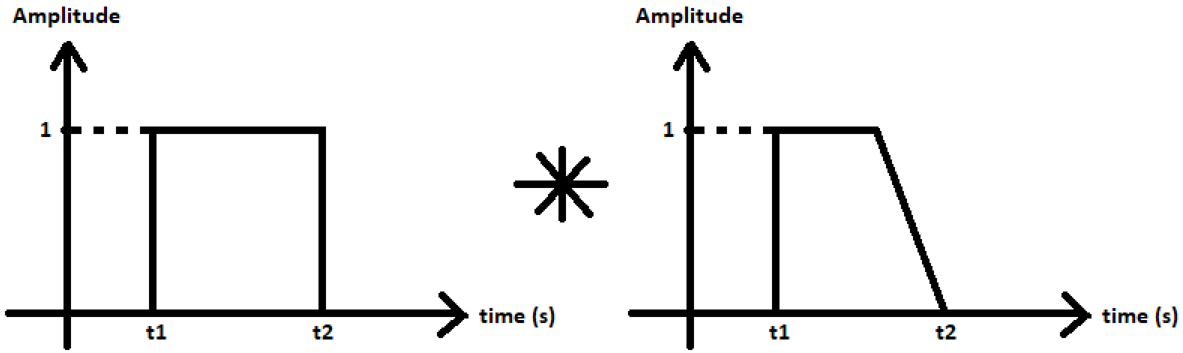
sinc(x)

Convolution Example

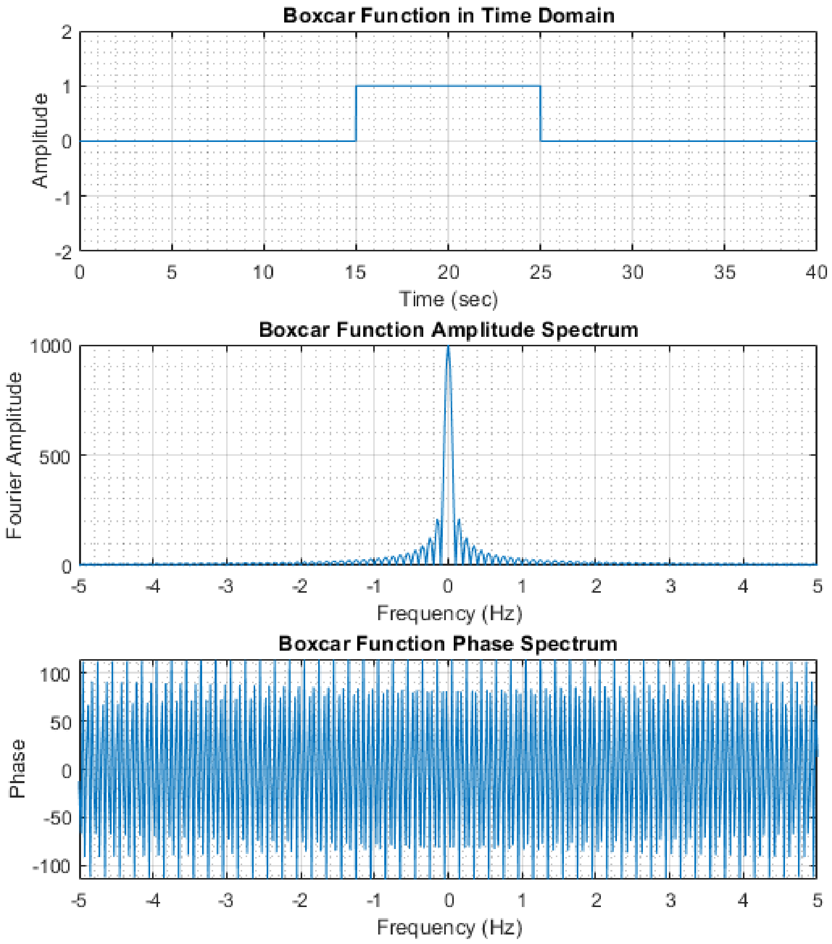
- Lets imagine the convolution of two boxcars…

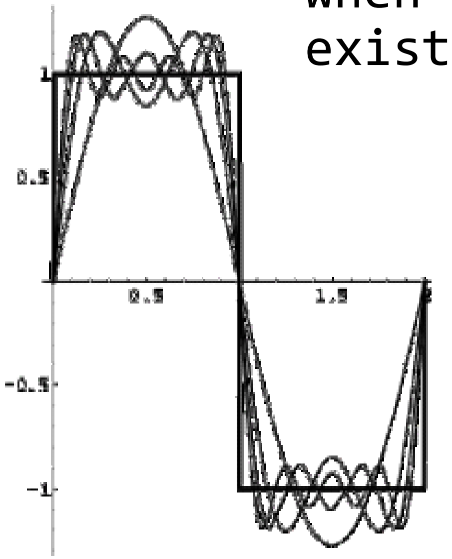


- Finally imagine a boxcar and an asymmetric function…



Gibb's Phenomenon





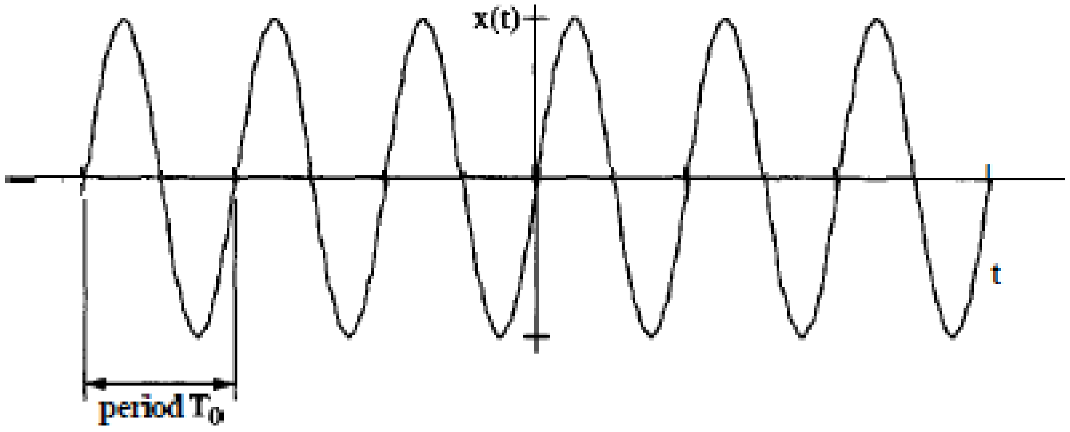
Rectangular signals can only be approximated by adding infinite cosine and sine functions. Gibbs phenomenon occurs when a sharp change exists in the signal.

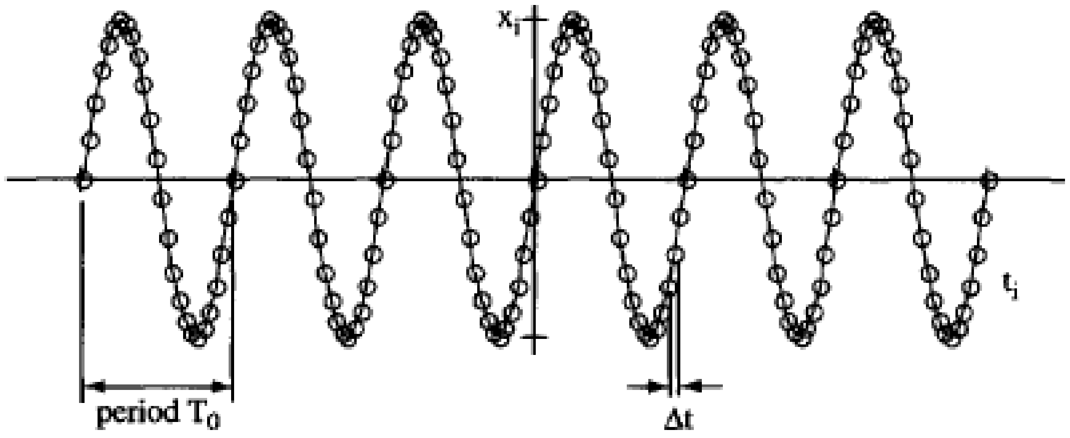
Gibb's phenomenon is even more emphasized in filtering…

Discrete Signals

So far, we were considering continuous (analytical) signals which were completely defined over a finite or infinite time domain.

However, computers work with signals defined by discrete data points (also called as *sample points* taken from the continuous signal).





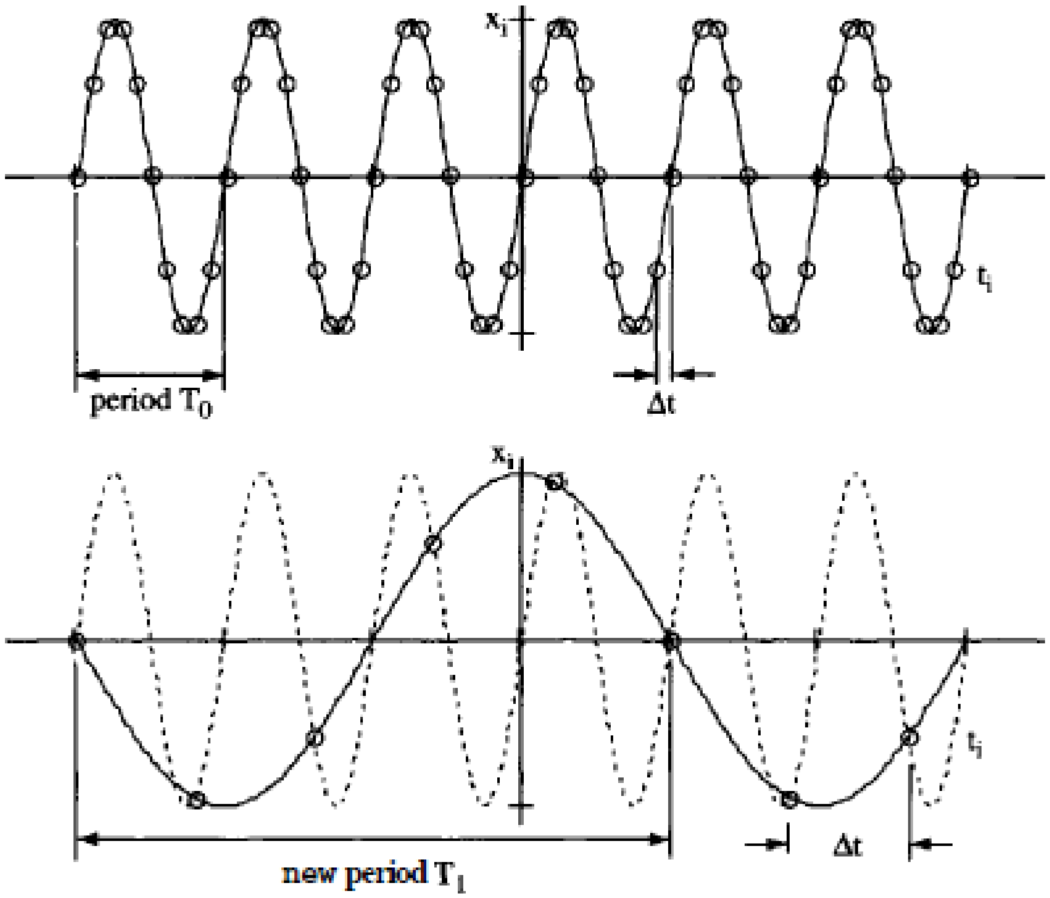
Nyquist Theorem

Nyquist theorem draws attention to a fundamental property of the discrete signals. It is only possible to properly identify the frequency of a discrete signal if the sampling frequency is higher than the Nyquist frequency.

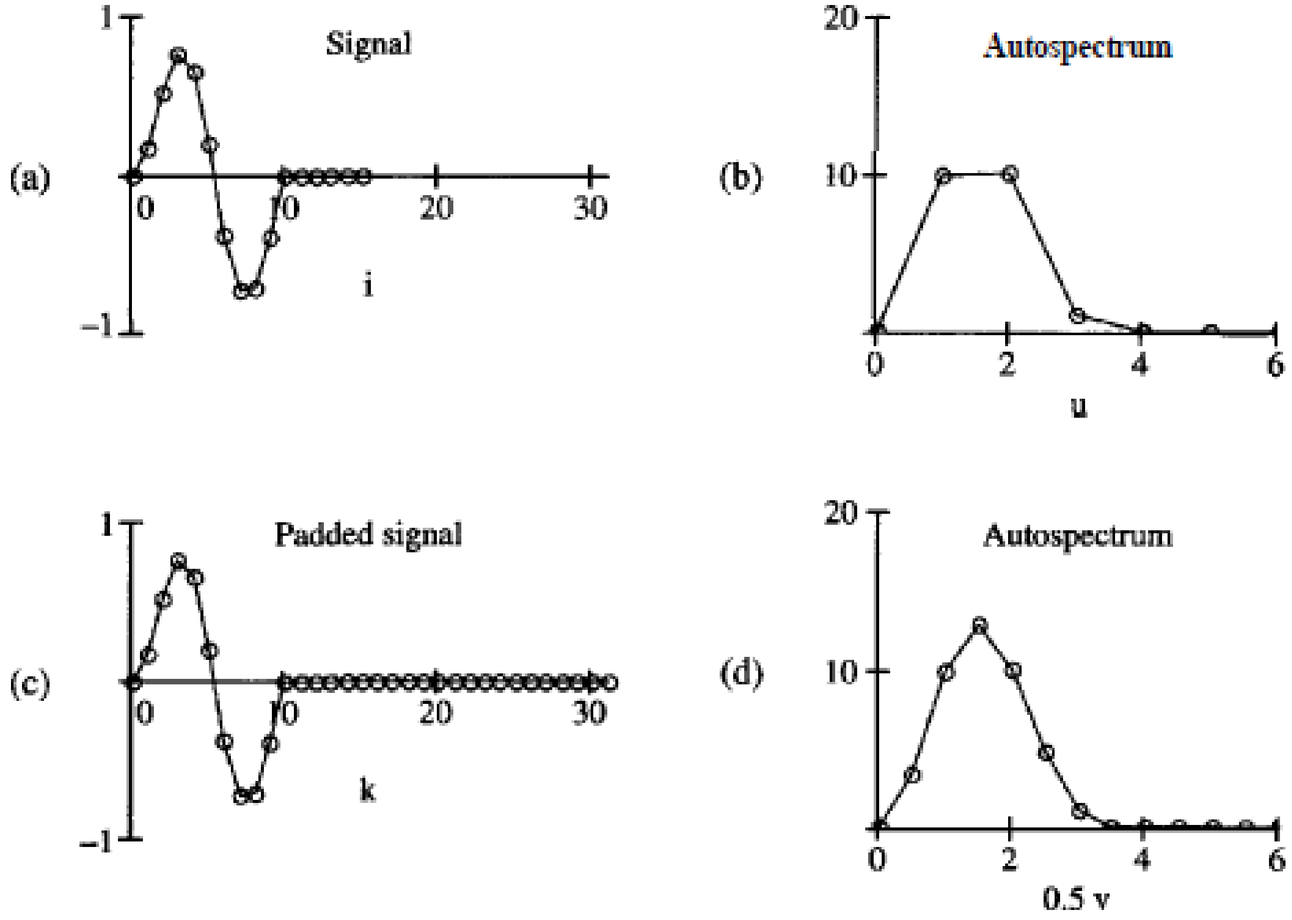
Sampling Frequency   Nyquist Frequency

$$f_{samp.} = \frac{1}{\Delta t} \quad > \quad f_{nq.} = \frac{2}{T_0}$$

In practice: Minimum 10 points per cycle



Padding



Frequency resolution of the signal is increased by increasing the signal duration. This is often done by adding zeros at the end of the signal.

Frequency Resolution:

$$\Delta_f = \frac{1}{\Delta_t * N}$$

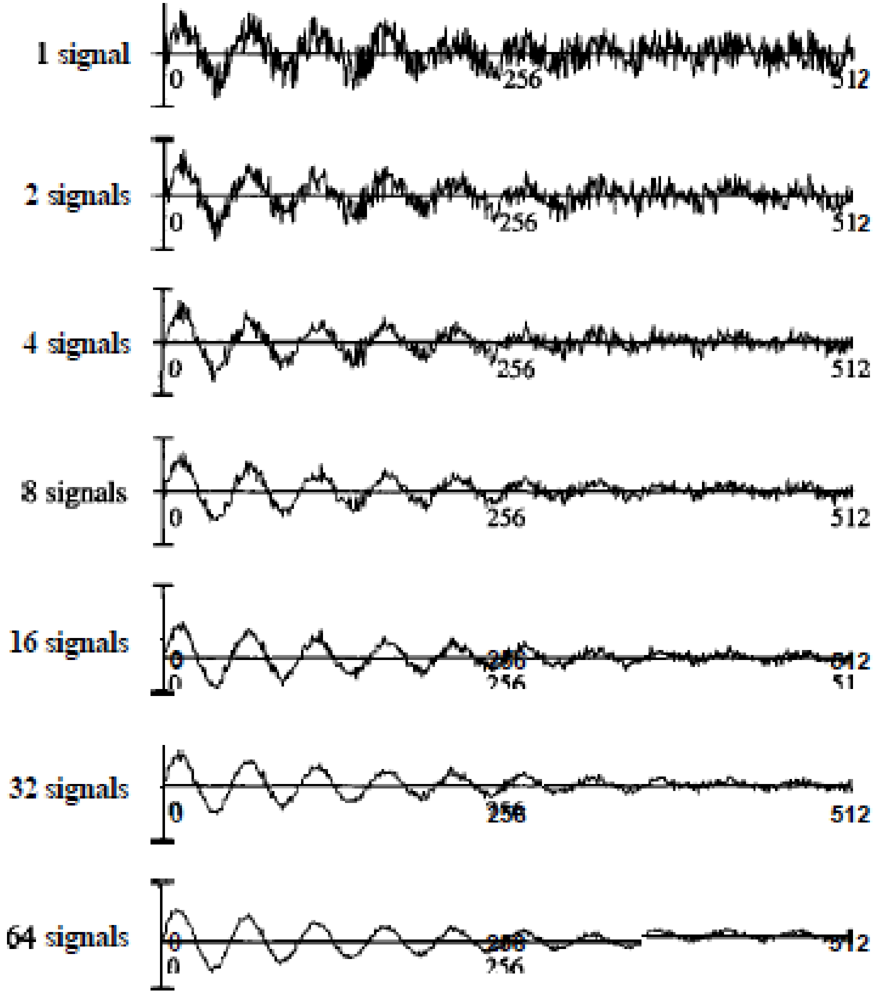
Noise

The high frequency distortion due to the environmental affects in the signal is often considered as noise.

Signal to Noise Ratio **(SNR)**

$$\mathbf{SNR} = \frac{\text{Amplitude of the Signal}}{\text{Amplitude of Noise}}$$

Repeating the recording several and *stacking* them together is a way to increase SNR.



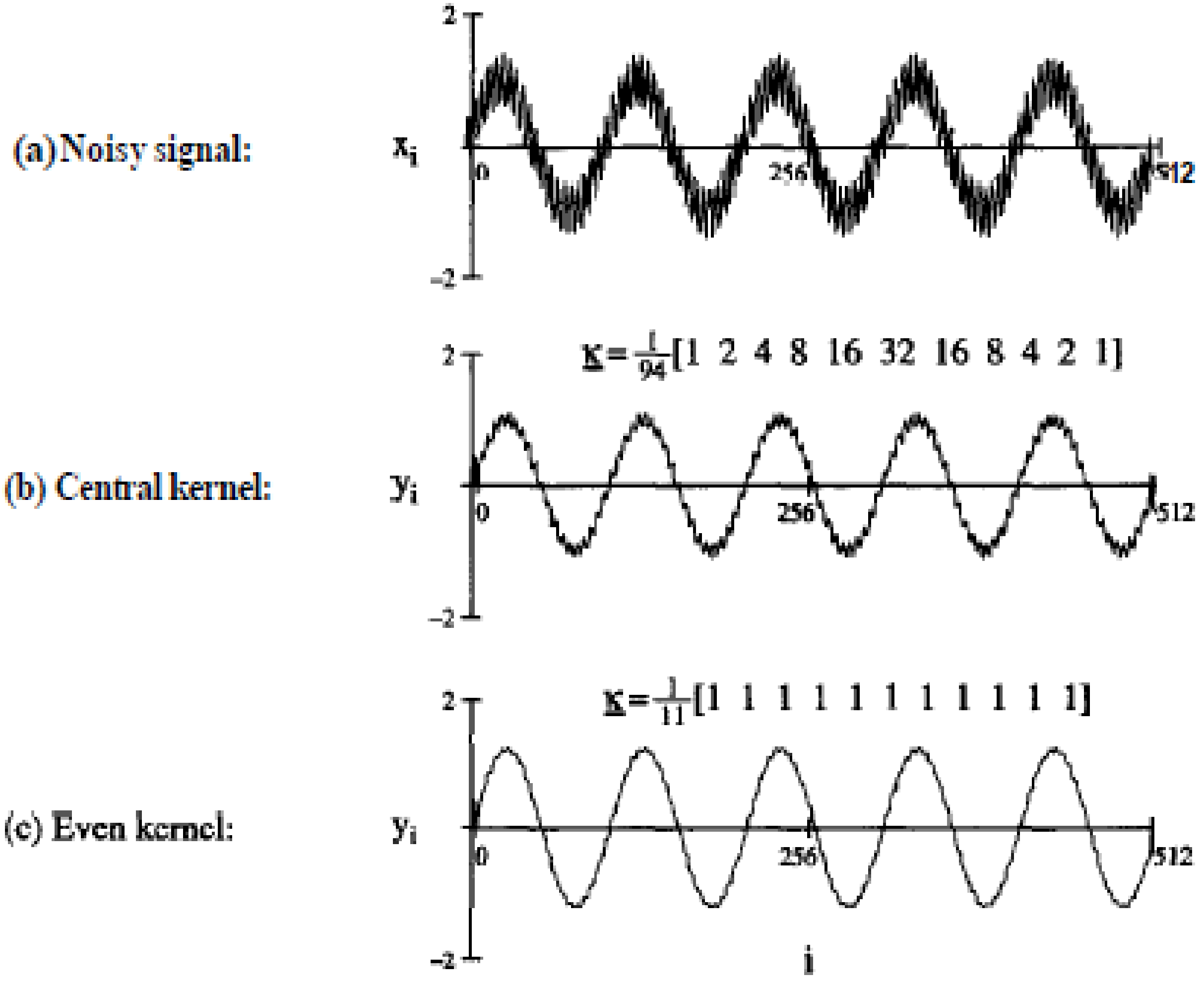
Moving Kernels

Moving kernel is an averaging time window sweeping the signal from start to end. Averaging the signal in local portions creates a “smoothed” signal, less distorted by noise.

The smoothed signal is obtained by the formula

$$y_i = \sum_{p=1}^{p=m} K_p . x\left(i - \frac{m-1}{2}\right) + p$$

Where Kernel length m, the weights K_p and x is the noisy signal



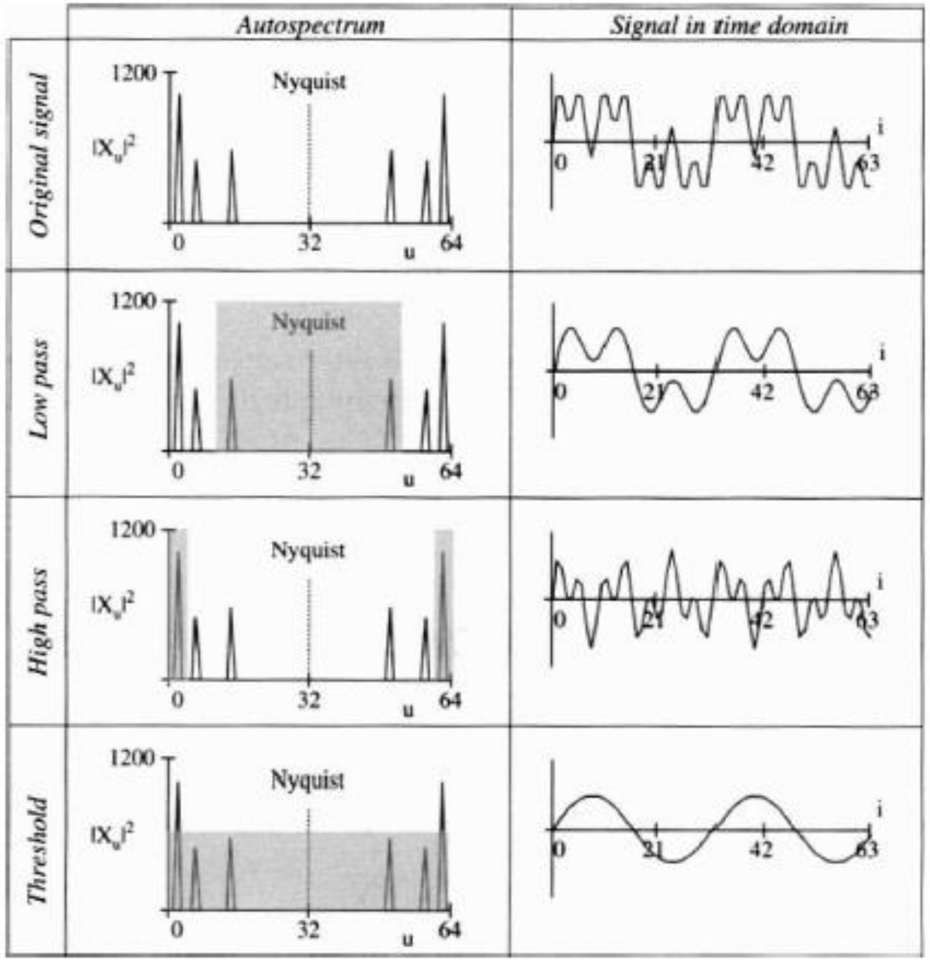
Filtering

In frequency domain, a filter is a window that is passing a band of frequency and blocking the rest. Filters are applied as a point by point multiplication with the amplitude and phase spectrum of the original signal.

Low Pass: Passes only the frequencies lower than the filter frequency.

High Pass: Passes only the frequencies higher than the filter frequency.

Band Pass: Passes only the frequencies in between the high and low frequency limits that are bounded by the filter.



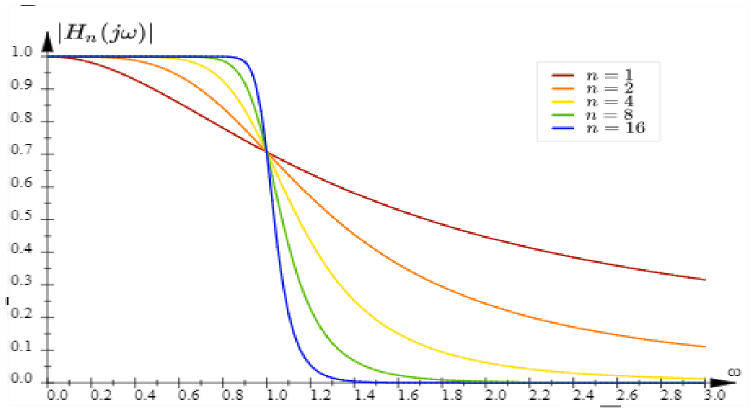
Ringing Artefact

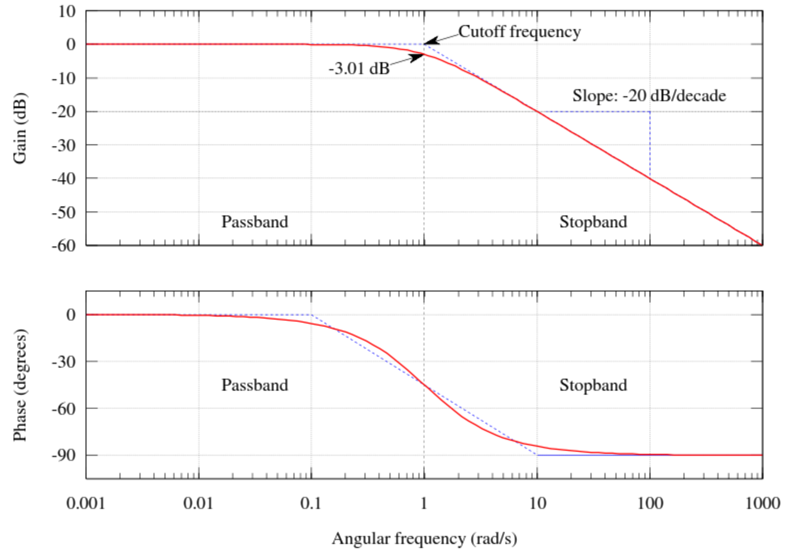
Butterworth Filter

Butterworth filter is a low pass filter with a cut off frequency of ω_c.

1^{st} order Transfer function:

$$|H_n(i\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

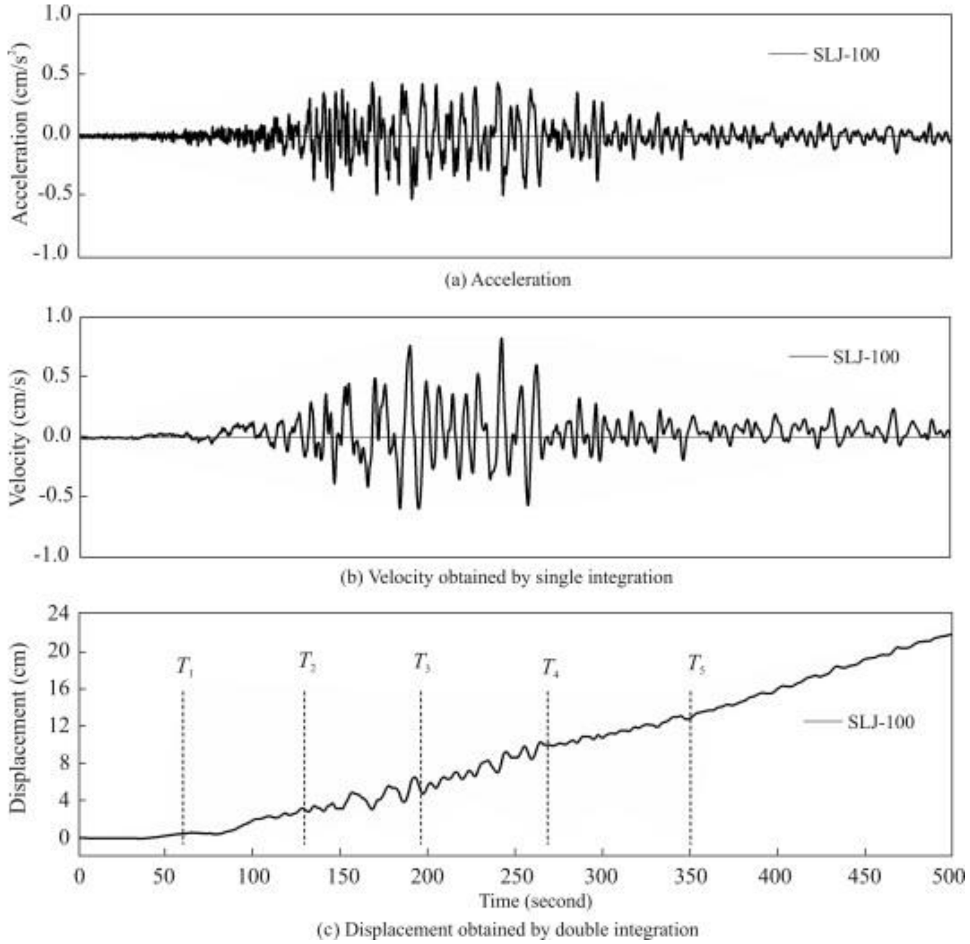




n	Factors of Polynomial $B_n(s)$
1	$(s+1)$
2	$(s^2+1.4142s+1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.7654s+1)(s^2+1.8478s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$
6	$(s^2+0.5176s+1)(s^2+1.4142s+1)(s^2+1.9319s+1)$
7	$(s+1)(s^2+0.4450s+1)(s^2+1.2470s+1)(s^2+1.8019s+1)$
8	$(s^2+0.3902s+1)(s^2+1.1111s+1)(s^2+1.6629s+1)(s^2+1.9616s+1)$
9	$(s+1)(s^2+0.3473s+1)(s^2+s+1)(s^2+1.5321s+1)(s^2+1.879s+1)$
10	$(s^2+0.3129s+1)(s^2+0.9080s+1)(s^2+1.4142s+1)(s^2+1.7820s+1)(s^2+1.9754s+1)$

← Pyramid of truth!

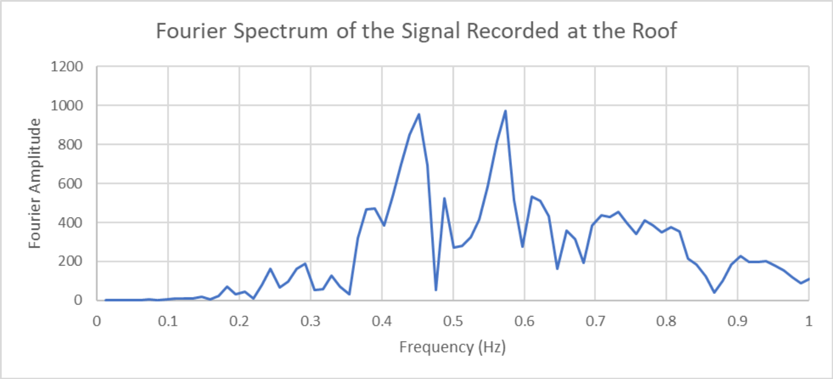
Baseline Correction

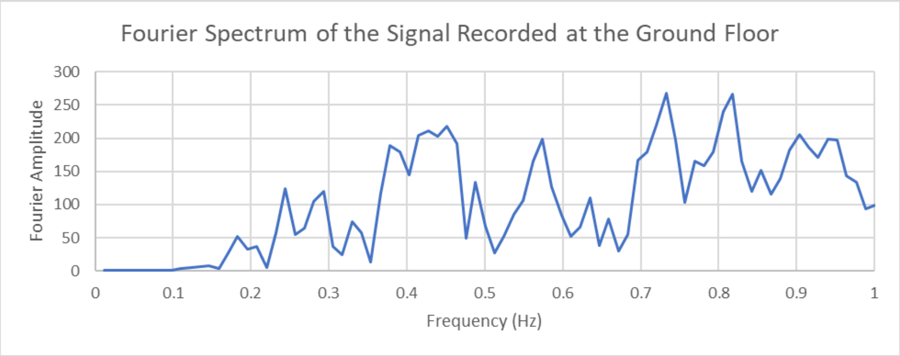


Baseline correction is needed when the signal is off the axis. Whether this is due to the instrument or the data, an upward shifted horizontal axis with a constant, may introduce significant errors especially after integration.

To remove the baseline, the mean of the signal is computed along the time domain and a linear/quadratic function is fitted. Then the fitted function is subtracted from the amplitude of the original signal.

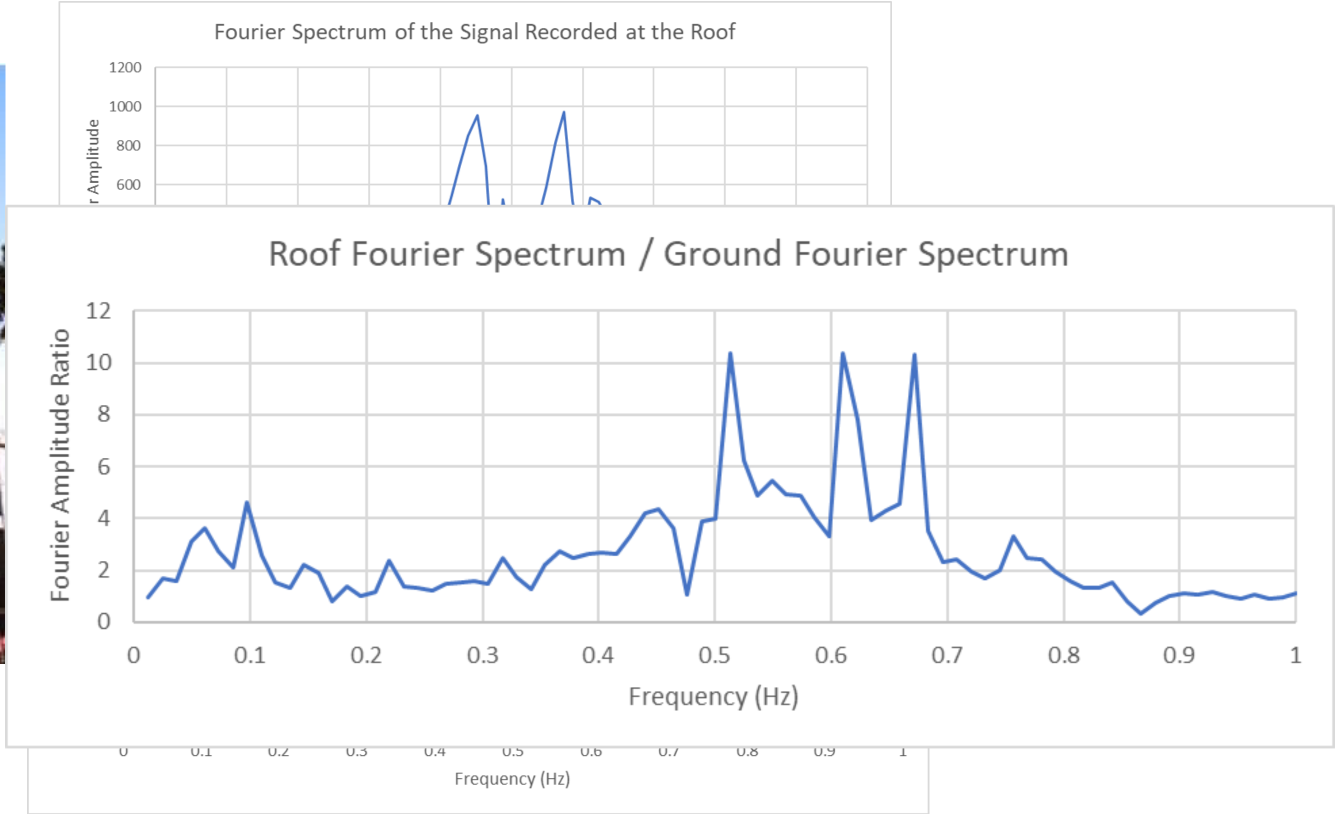
Van Nuys Building





PEER Testbed, 7 Storey Hotel in Van Nuys, California

Van Nuys Building



Transfer Function →

Used of the in structural health monitoring!

Further Reading

- Also as a reference for this tutorial notes:

- Santamarina, J.C. & Fratta, D. (2005) *Discrete Signals and Inverse Problems. An Introduction for Engineers and Scientists*. Published by John Wiley & Sons, Ltd, West Sussex, England