Quick review of some fundamentals of Fourier analysis

Here is a super quick summary of some fundamental facts of Fourier analysis. Have a quick look to make sure they're familiar.

- Fourier Series (possibly in several variables)

Let $f : \mathbb{R}^d \to \mathbb{C}$ be periodic in each variable with period 1.
Definition: for $n \in \mathbb{Z}^d$, $\hat{f}(n) = \int_{[0,1]^d} f(x) e^{-2\pi i n \cdot x} dx$.

Fourier inversion theorem: If f is C^{n+1}, then $f(x) = \sum_{n \in \mathbb{Z}^d} \hat{f}(n) e^{2\pi i n x}$, and the sum converges pointwise.

Convolution: For functions f and g as above, $f * g(x) = \int_{[0,1]^d} f(y) g(x-y) dy$.

Parseval's Identity: $\int_{[0,1]^d} |f(x)|^2 dx = \sum_{n \in \mathbb{Z}^d} |\hat{f}(n)|^2$.

If f is in $L^2([0,1]^d)$, then $\hat{f}(n)$ is well-defined, and f is the limit in L^2 of the partial sums $\sum_{|n| \leq N} \hat{f}(n) e^{2\pi i n x}$.

- Fourier Transform

Let f be a function on $\mathbb{R}^d$. (So $x, \xi \in \mathbb{R}^d$.)
We define $\hat{f}(\xi) = \int_{\mathbb{R}^d} f(x) e^{-2\pi i x \cdot \xi} dx$.
Fourier inversion: If f is Schwartz, $f(x) = \int_{\mathbb{R}^d} \hat{f}(\xi) e^{2\pi i x \cdot \xi} d\xi$.
Plancherel Theorem: If f is Schwartz, $\int_{\mathbb{R}^d} |f(x)|^2 dx = \int_{\mathbb{R}^d} |\hat{f}(\xi)|^2 d\xi$.
Since the Schwartz functions are dense in L^2, the Fourier transform extends to an isomorphism of $L^2(\mathbb{R}^d)$.

- Main Formulas about the Fourier Transform

a. $(\partial_j f)^\wedge(\xi) = 2\pi i \xi_j \hat{f}(\xi)$.
b. $(f * g)^\wedge(\xi) = \hat{f}(\xi)\hat{g}(\xi)$.
c. $(f \cdot g)^\wedge(\xi) = (\hat{f} * \hat{g})(\xi) = \int_{\mathbb{R}^d} \hat{f}(\omega)\hat{g}(\xi - \omega) d\omega$.