Engineering Seismology and Seismic Hazard – 2019

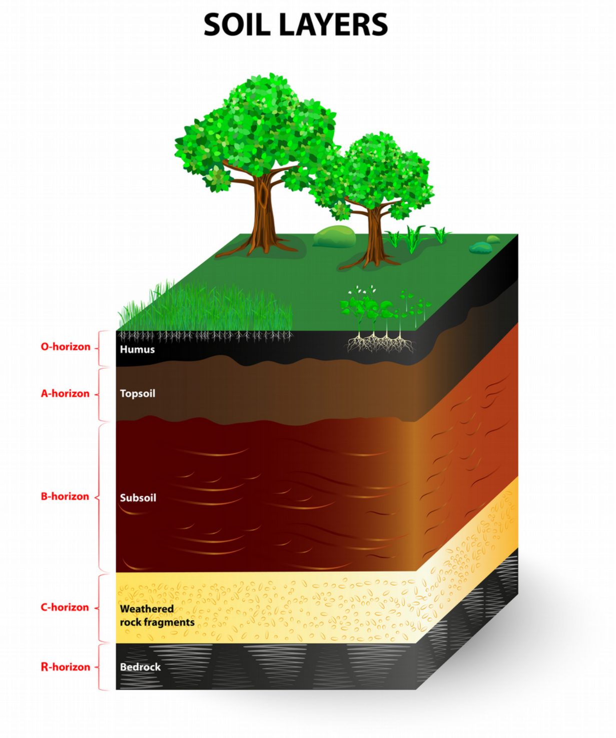
Lecture 7

Waves in Complex Media

Valerio Poggi
Seismological Research Center (CRS)
National Institute of Oceanography and Applied Geophysics (OGS)

Heterogenous Media

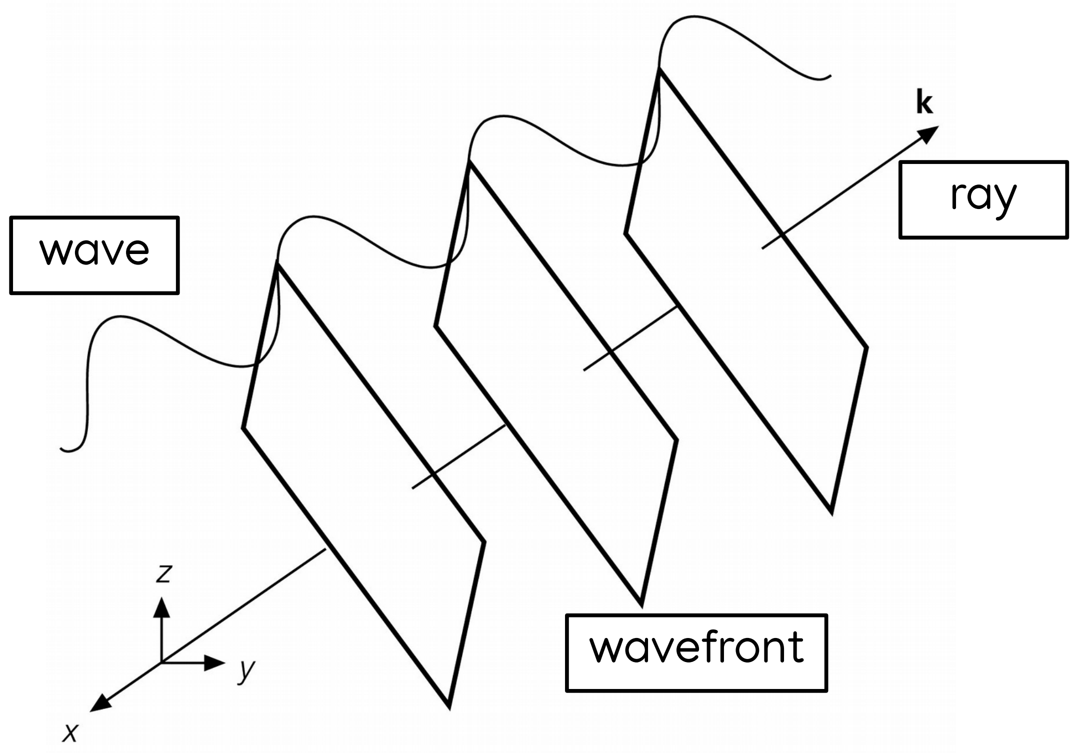
So far, we have only considered waves traveling into an homogeneous infinite medium. However, earth is intrinsically heterogenous (at all scales!)



We now consider the case of waves interacting with actual physical boundaries, such as:
1) interfaces between media of different physical properties
2) the free surface

The Concept of Ray

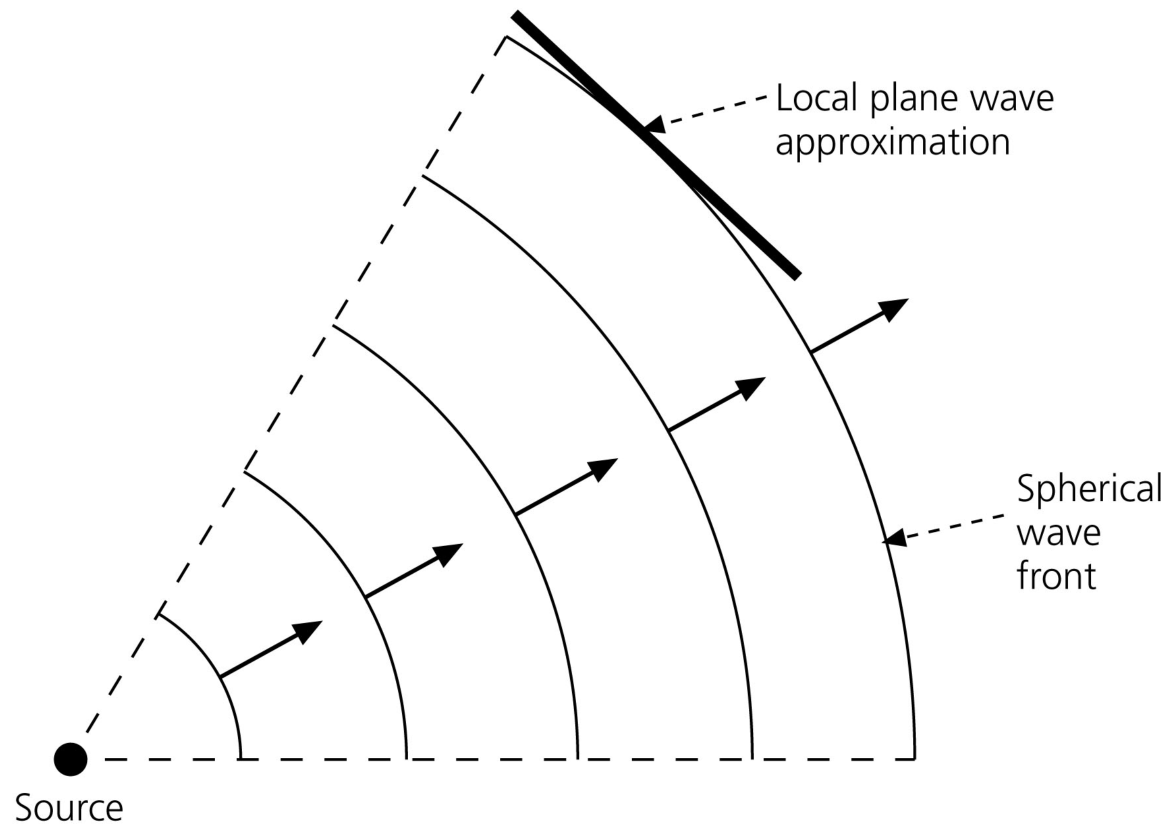
A ray is a geometrical artifice to describe the direction of propagation of a wave at all instants along its path. It is locally defined as the normal to the wavefront.



Note: Ray theory is high frequency approximation, which means the size of heterogeneities and variation of mechanical properties are far larger than the wavelength

Planar and Spherical Wavefronts

It must be kept in mind that planar wavefronts do not actually exist in reality.
A plane wave is mathematically convenient way to locally approximate a spherical wavefront, e.g. originated by a point source.

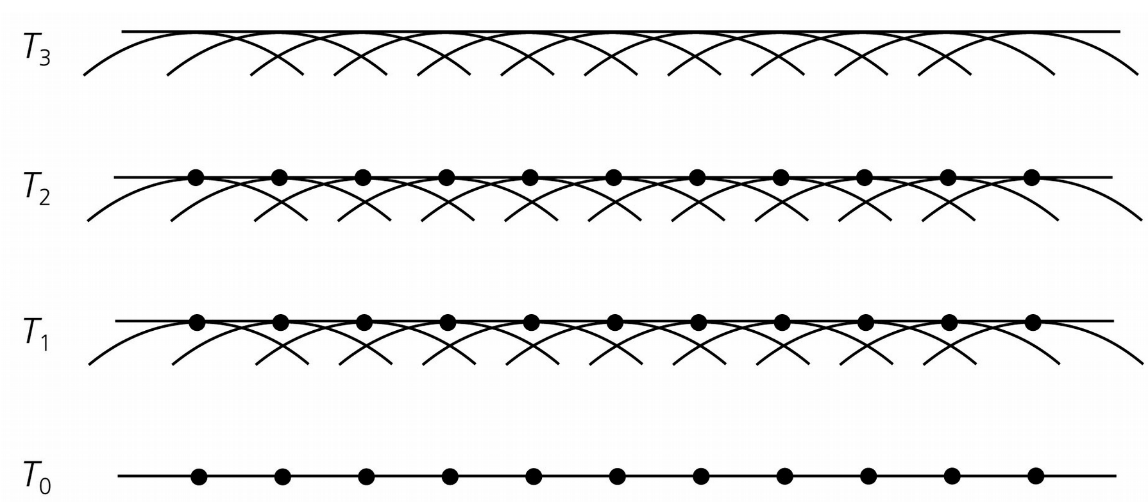


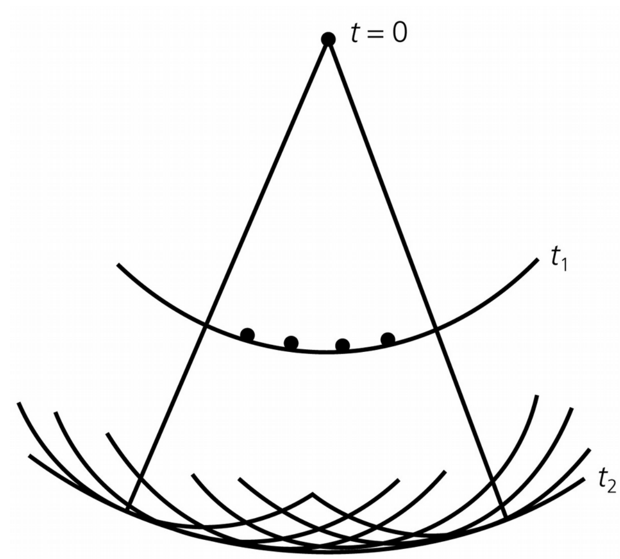
Huygen's Principle

Every point on a wavefront is itself the source of spherical wavelets.

The sum of these spherical wavelets forms the new wavefront after a time Δt.

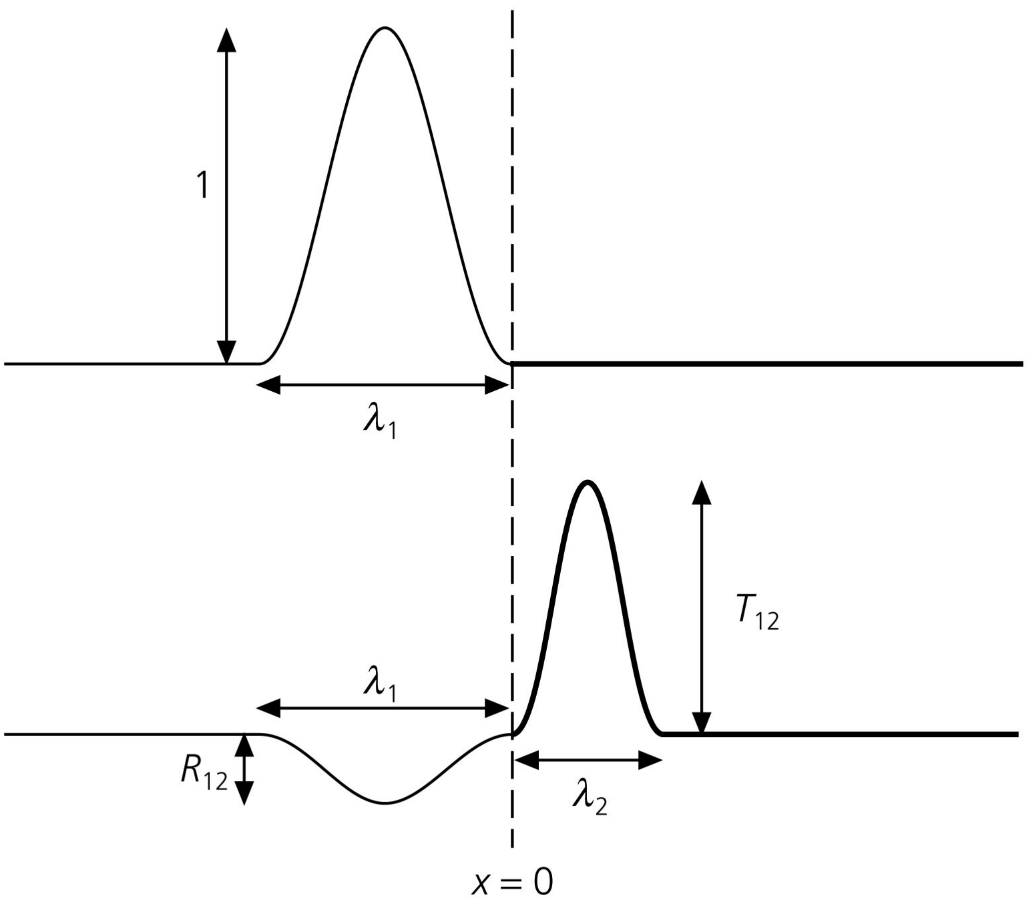
This is nonetheless a purely geometrical interpretation.





Reflection and Transmission

As for the case of optics, a ray hitting an interface between media with different elastic properties will be both reflected and refracted.

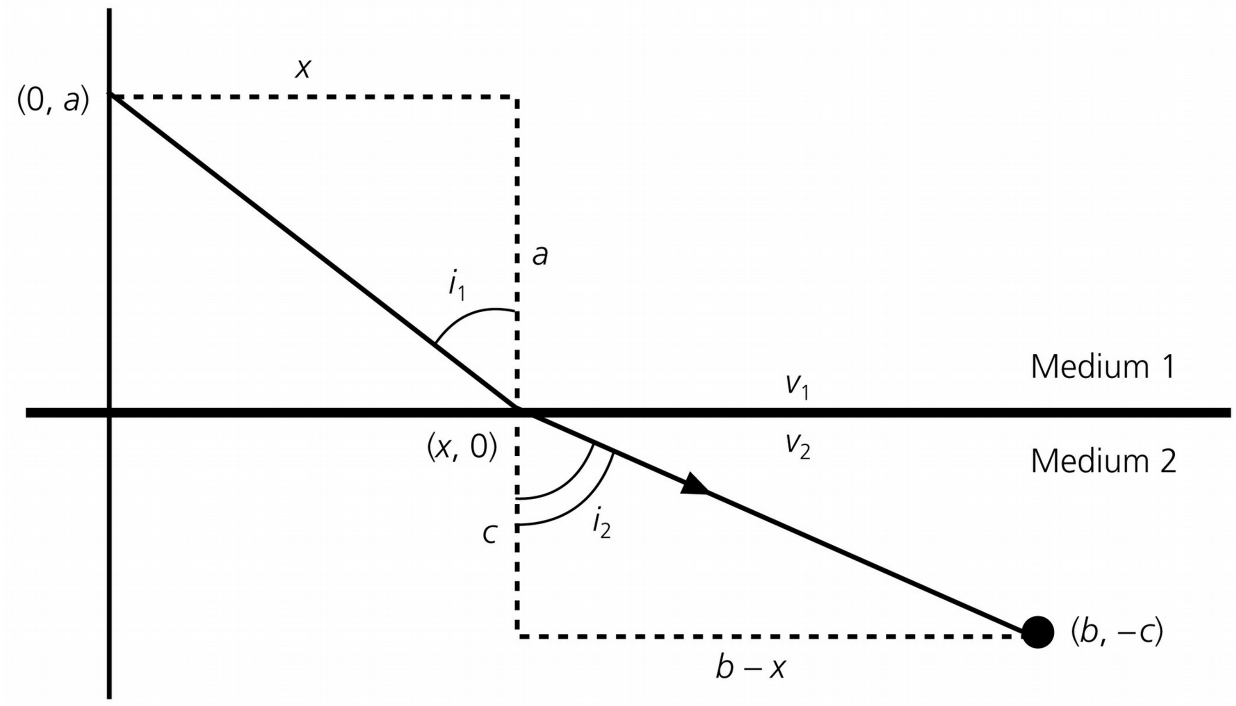


Energy will be partitioned between the reflected and refracted components

Transmission at Interface

The path crossing the interface between media of different properties can be geometrically formalized as:

$$T(x) = \frac{\sqrt{a^2 + x^2}}{v_1} + \frac{\sqrt{(b - x)^2 + c^2}}{v_2}$$



Fermat's Principle and Snell's Law

The Fermat's principle states that a ray path between two points is that for which travel time is minimum.
The previous travel time can then be minimized by differentiation:

$$\frac{dT(x)}{dx} = \frac{x}{v_1\sqrt{a^2+x^2}} - \frac{b-x}{v_2\sqrt{(b-x)^2+c^2}} = 0$$

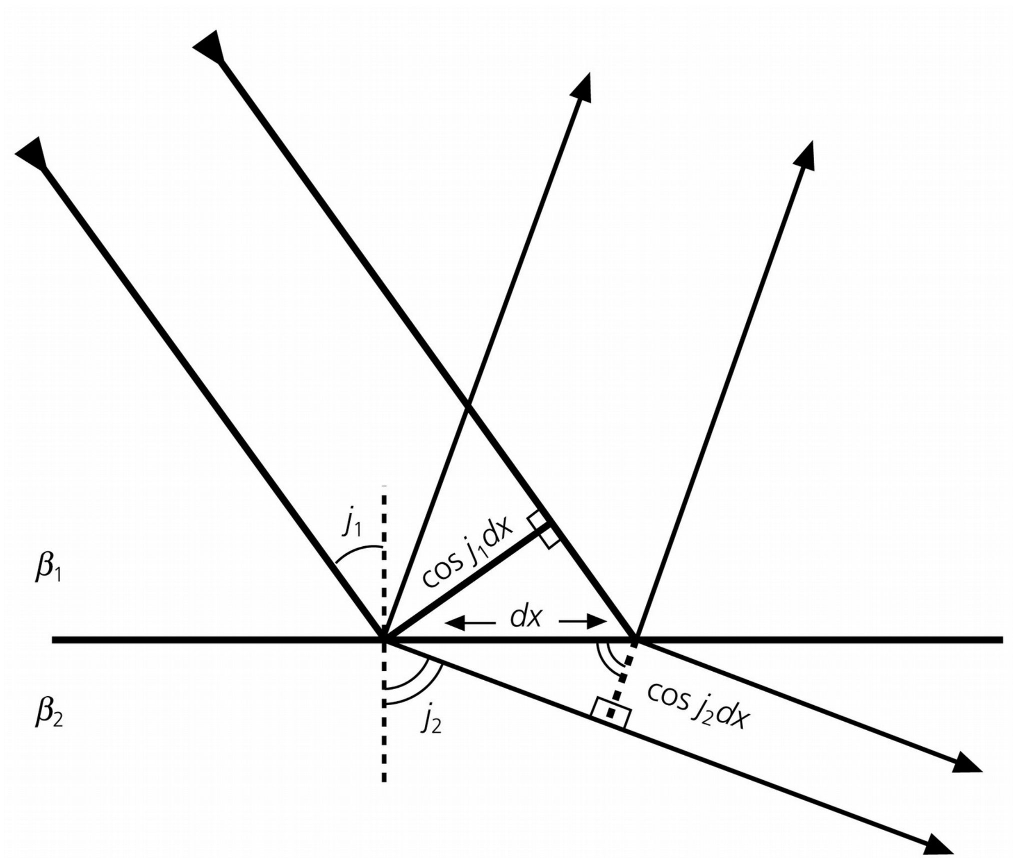
$$\frac{dT(x)}{dx} = \frac{\sin i_1}{v_1} - \frac{\sin i_2}{v_2} = 0$$

$$\frac{\sin i_1}{v_1} = \frac{\sin i_2}{v_2} = p$$

This last is the Snell's law, which can be applied for transmission and reflection. p in an invariant quantity named ray parameter.

Snell's Law from Huygen's Principle

Note that same conclusions can be found considering that the wavefront propagation time in medium 1 is equal to that in medium 2 (following Huygen's principle)

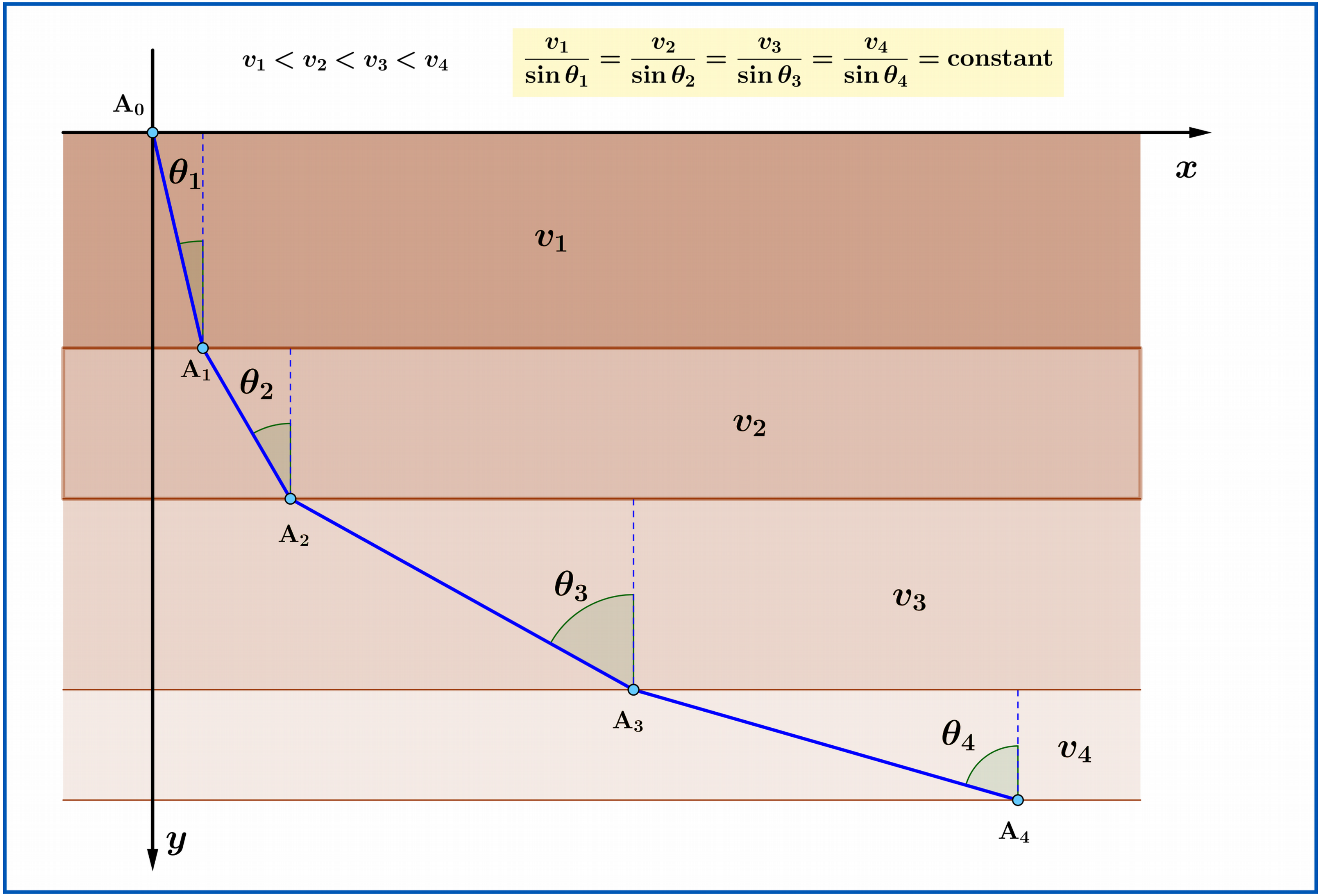


$$t_1 = t_2$$

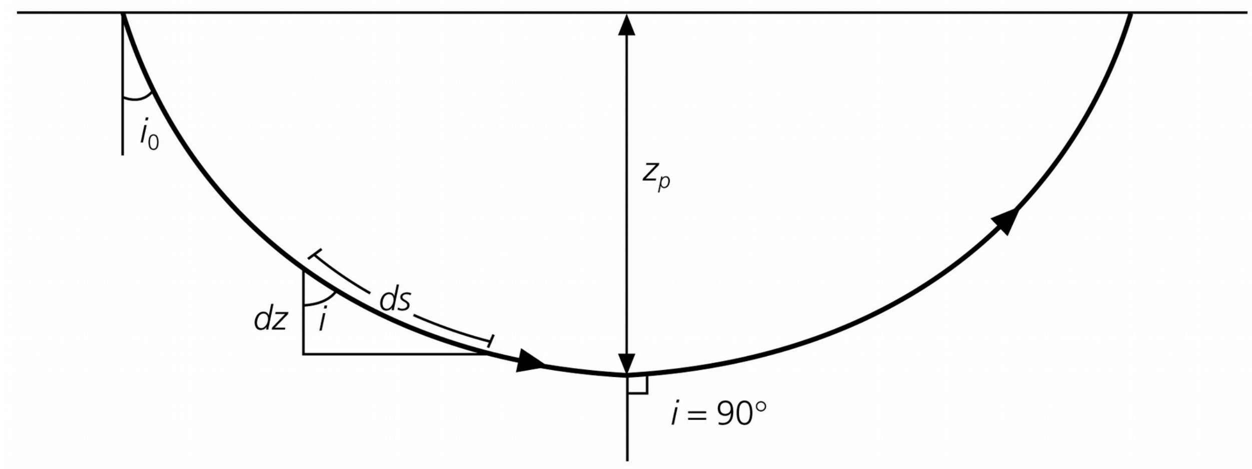
$$t_1 = \frac{\sin i_1 \, dx}{v_1}$$

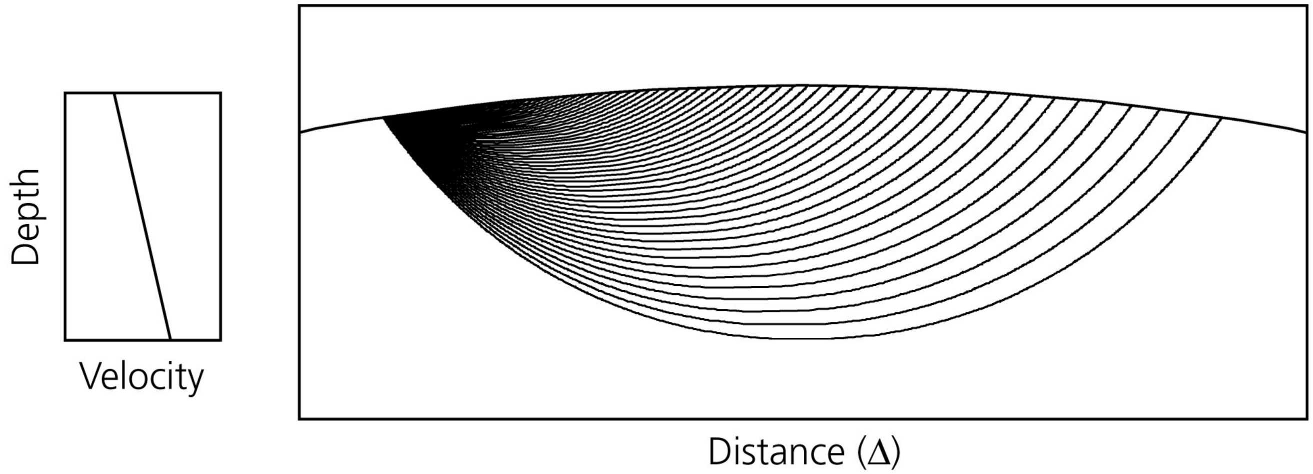
$$t_2 = \frac{\sin i_2 \, dx}{v_2}$$

Transmission in Multiple Layers



Effect of Velocity Gradients

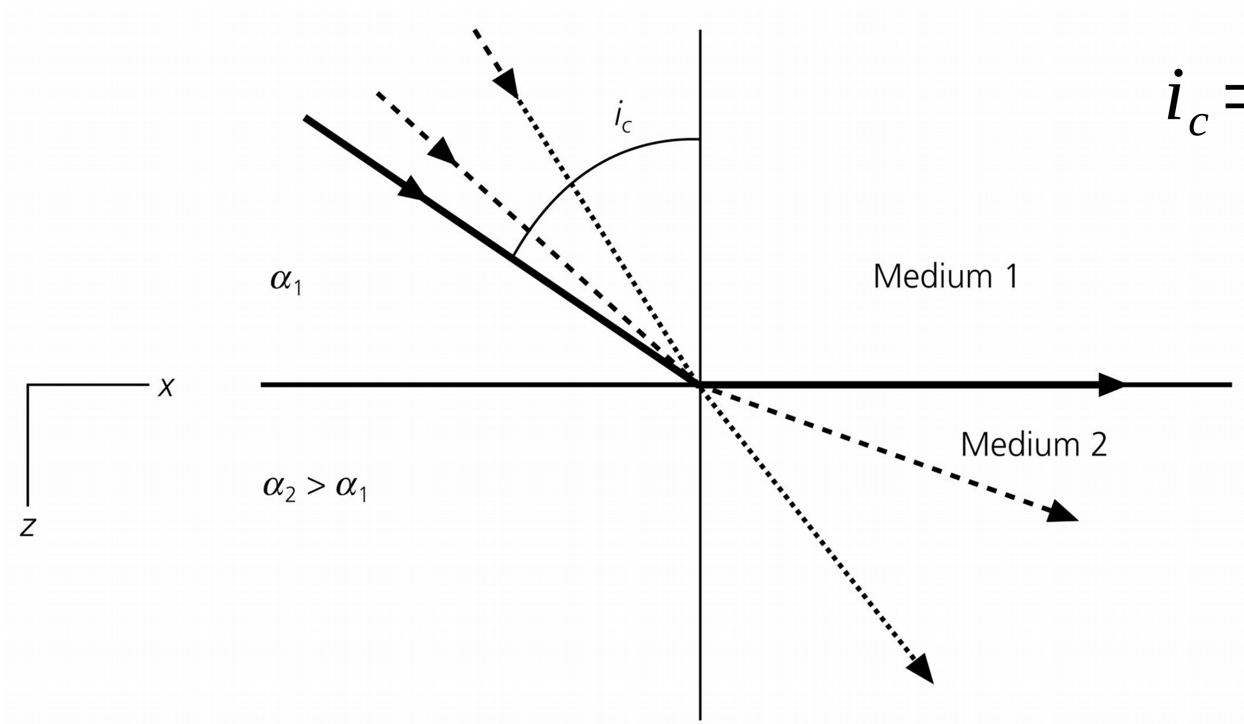




Critical Angle

In a particular case, an incidence angle can lead to transmitted angle emerging parallel to the interface. In such a case, the incidence angle is called "critical".

For angles larger than the critical, reflection is total.

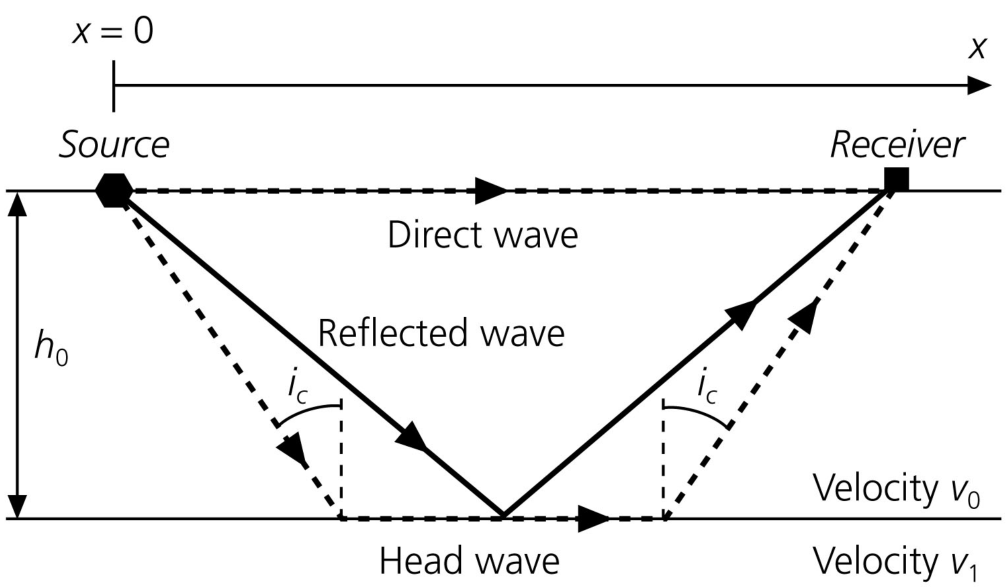


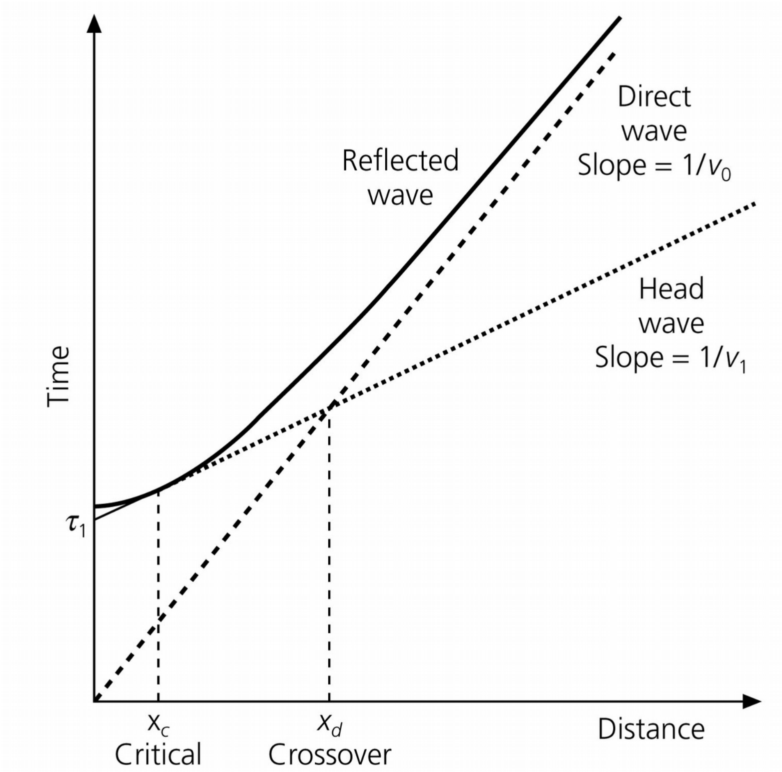
$$i_c = \arcsin\left(\frac{v_1}{v_2}\right)$$

Critical Refraction

The wave refracted at critical angle runs parallel to the interface, but can emerge any time following a “reverse path”.

We speak more properly of Head Waves





P-SV and SH Decomposition

Right now we have considered only a kind of "acoustic" propagation. What would happen in case of elastic waves?

First, we recall that displacement can be expressed as:

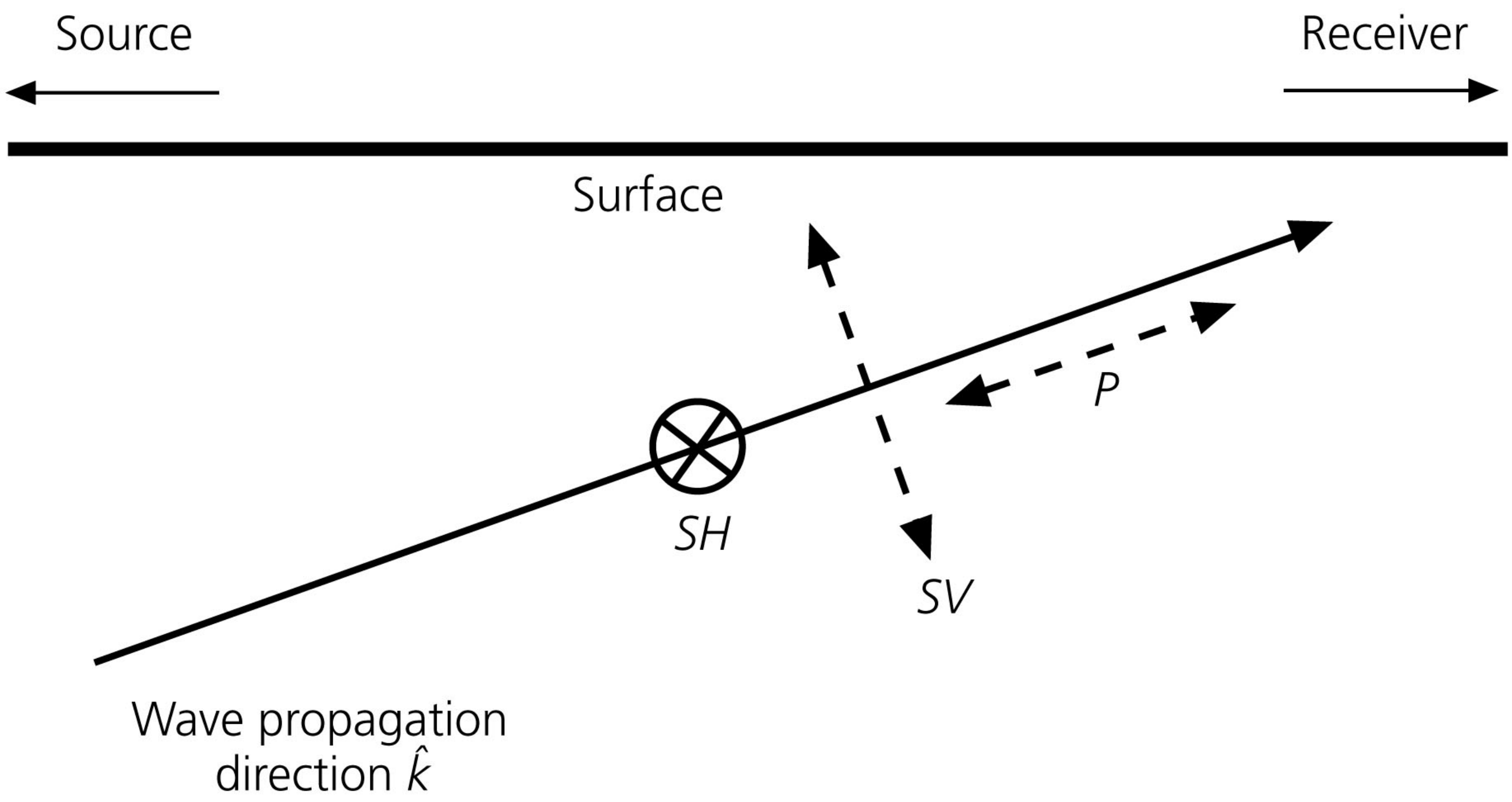
$$\vec{u} = \vec{u}_P + \vec{u}_S = \nabla \phi + \nabla \times \vec{\psi}$$

where:

$$\vec{u}_P = \frac{\delta \phi}{\delta x_1} \hat{i} + \frac{\delta \phi}{\delta x_2} \hat{j} + \frac{\delta \phi}{\delta x_3} \hat{k}$$

$$\vec{u}_S = \left(\frac{\delta \psi_3}{\delta x_2} - \frac{\delta \psi_2}{\delta x_3} \right) \hat{i} + \left(\frac{\delta \psi_1}{\delta x_3} - \frac{\delta \psi_3}{\delta x_1} \right) \hat{j} + \left(\frac{\delta \psi_2}{\delta x_1} - \frac{\delta \psi_1}{\delta x_2} \right) \hat{k}$$

P–SV and SH Polarization



P–SV and SH Decomposition

If the waves propagates in the plane x1x3, then derivatives to x2 will vanish, giving displacements as:

$$\vec{u}=\left(\frac{\delta\,\phi}{\delta\,x_1}-\frac{\delta\,\psi_2}{\delta\,x_3}\right)\hat{i}+\left(\frac{\delta\,\psi_1}{\delta\,x_3}-\frac{\delta\,\psi_3}{\delta\,x_1}\right)\hat{j}+\left(\frac{\delta\,\phi}{\delta\,x_3}+\frac{\delta\,\psi_2}{\delta\,x_1}\right)\hat{k}$$

P Sv

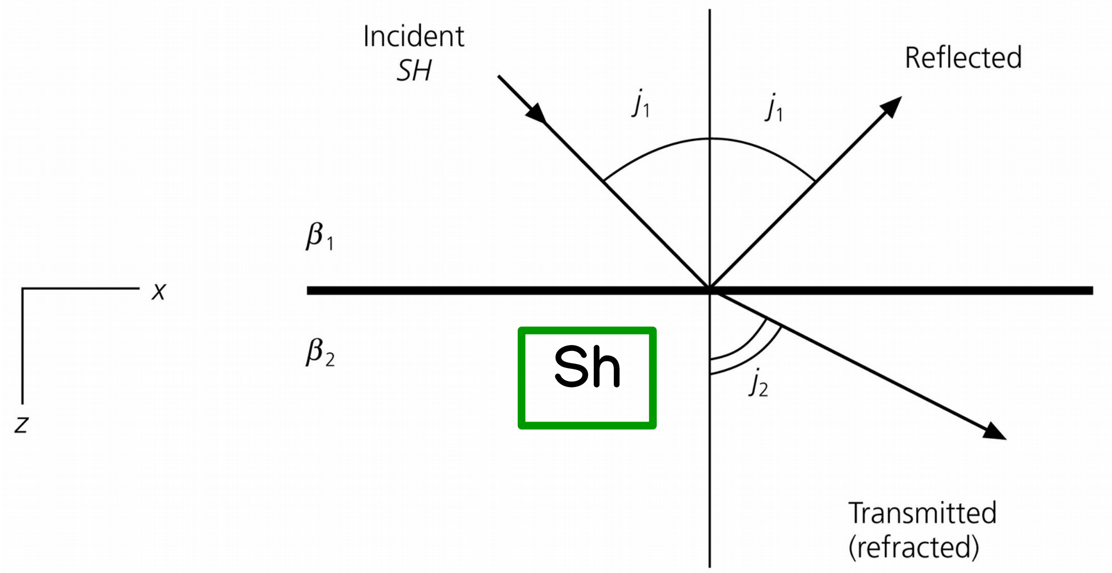
Sh

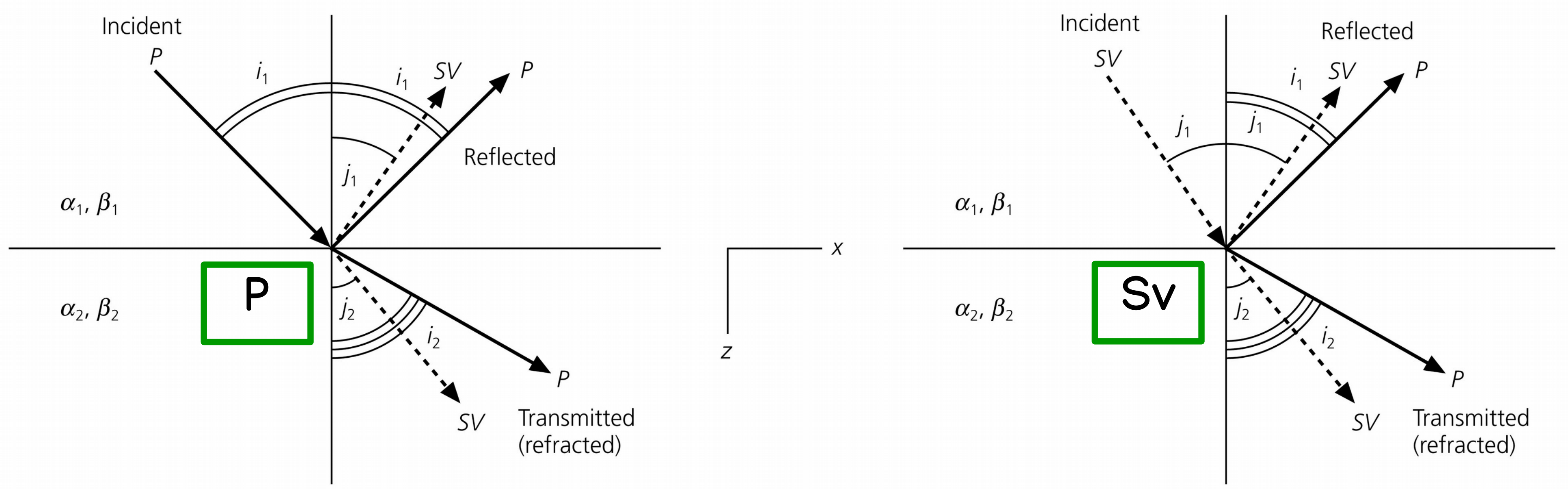
P Sv

$$\vec{u}=\vec{u}_P+\vec{u}_S=\vec{u}_P+\vec{u}_{Sv}+\vec{u}_{Sh}$$

It is then clear that P and Sv components are related, while Sh is independent.

P–S Wave Conversion





P–S Wave Conversion

P is invariant through all wave components, which means:

$$p = \frac{\sin i_1^\alpha}{\alpha_1} = \frac{\sin i_2^\alpha}{\alpha_2} = \frac{\sin i_1^\beta}{\beta_1} = \frac{\sin i_2^\beta}{\beta_2}$$

$$\alpha_1 > \alpha_2 \Rightarrow i_1^\alpha < i_2^\alpha$$

$$\beta_1 > \beta_2 \Rightarrow i_1^\beta < i_2^\beta$$

$$\alpha_1 > \beta_2 \Rightarrow i_1^\alpha < i_2^\beta$$

$$\alpha_2 > \beta_2 \Rightarrow i_2^\alpha < i_2^\beta$$

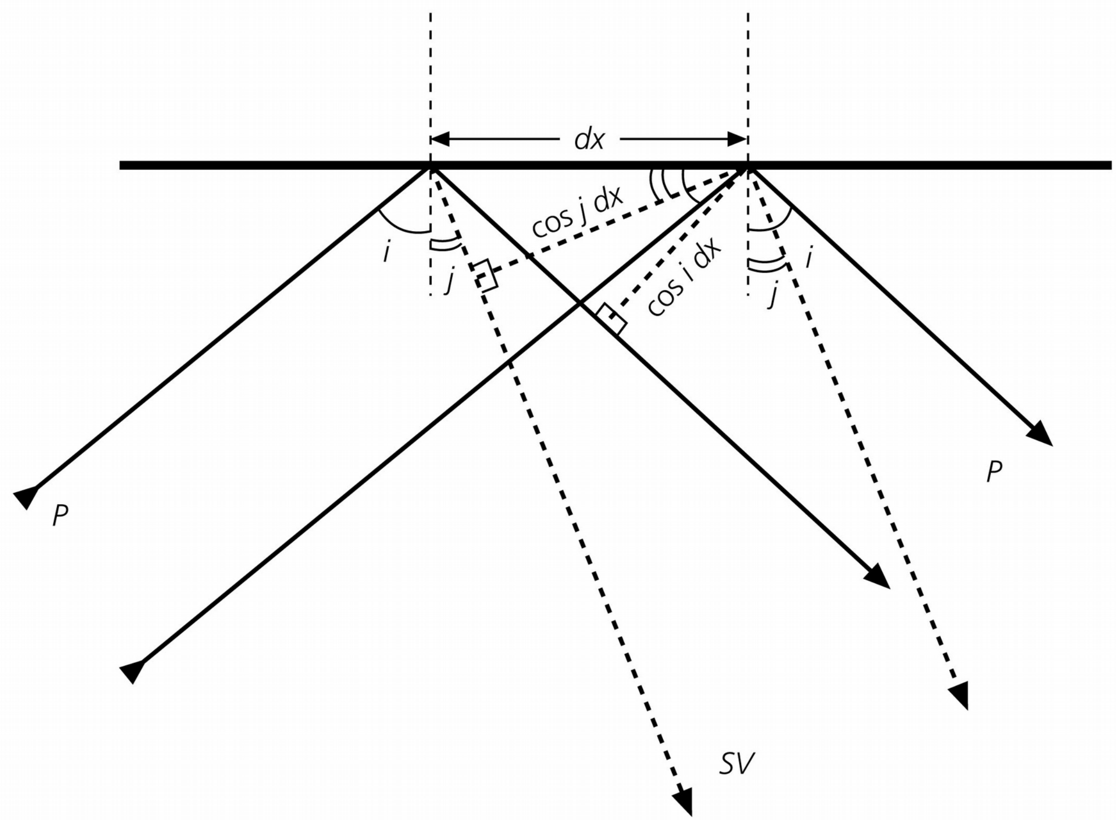
$$\alpha_1 > \beta_1 \Rightarrow i_1^\alpha < i_1^\beta$$

$$\alpha_2 > \beta_1 \Rightarrow i_2^\alpha < i_1^\beta$$

Free Surface Reflection

Same considerations will be applied to the free (ground) surface reflection.

Snell's Law is applicable also in this case. For an incident P-wave, a specular P-wave will be reflected, together with an S-wave with a smaller propagation angle.



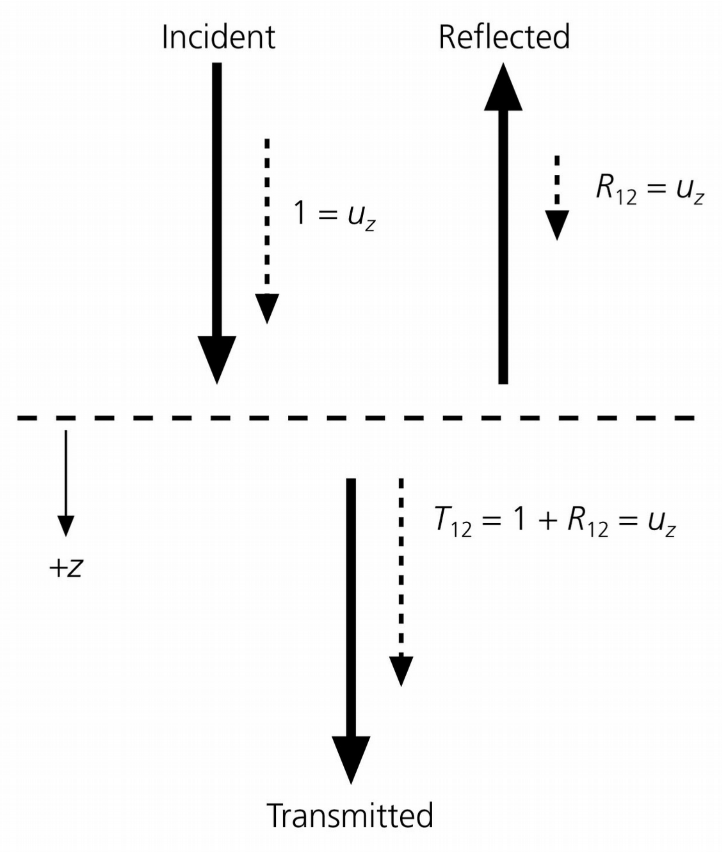
Amplitude Partitioning Coefficients

Reflection and transmission coefficients can be derived by solving boundary conditions (continuity of displacement and stress) at the interface.

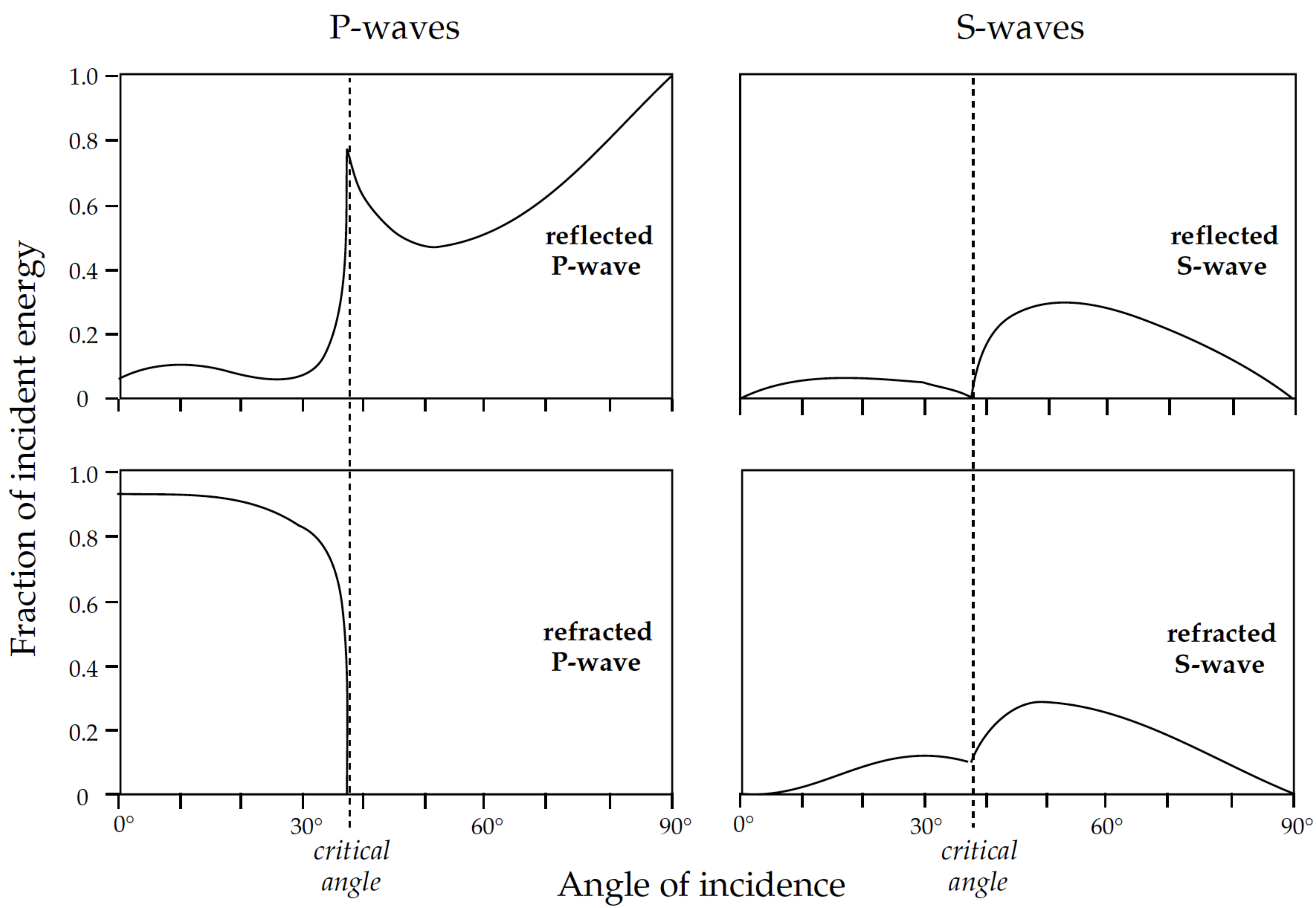
For the case of SH waves:

$$\frac{A_T}{A_I} = \frac{2\rho_1\beta_1\cos i_1}{\rho_1\beta_1\cos i_1 + \rho_2\beta_2\cos i_2}$$

$$\frac{A_R}{A_I} = \frac{\rho_1\beta_1\cos i_1 - \rho_2\beta_2\cos i_2}{\rho_1\beta_1\cos i_1 + \rho_2\beta_2\cos i_2}$$

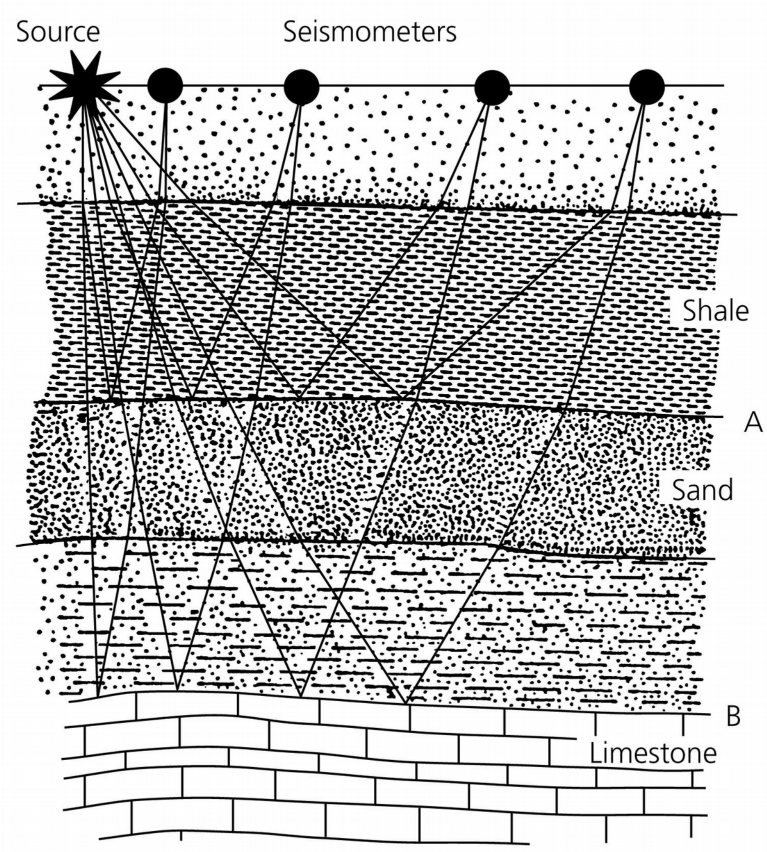


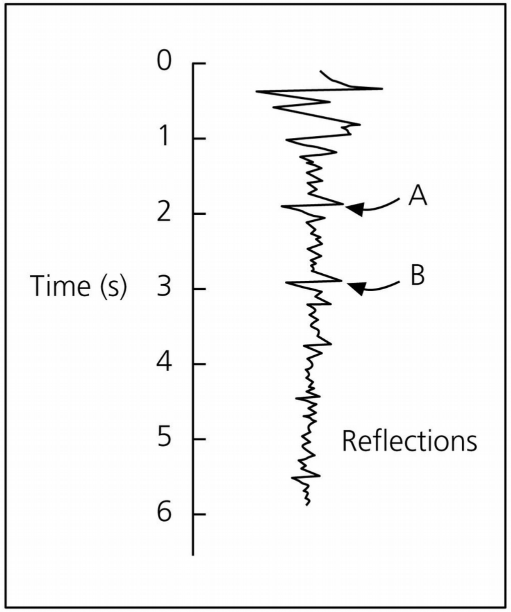
Partition at Critical Angle



Wave Arrivals on Seismograms

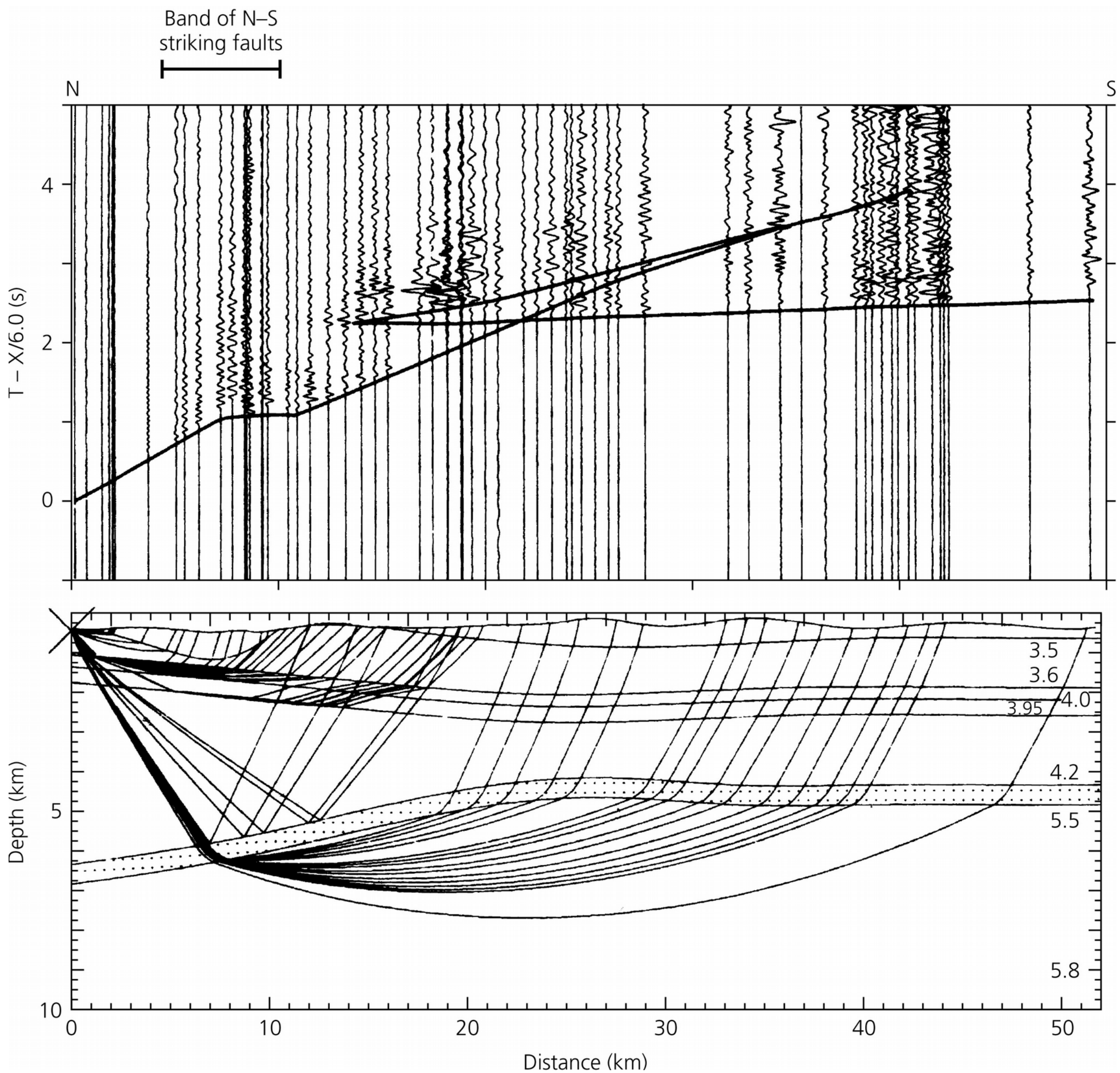
Arrival time of reflected or refracted phases is located on a seismogram though a procedure called phase picking





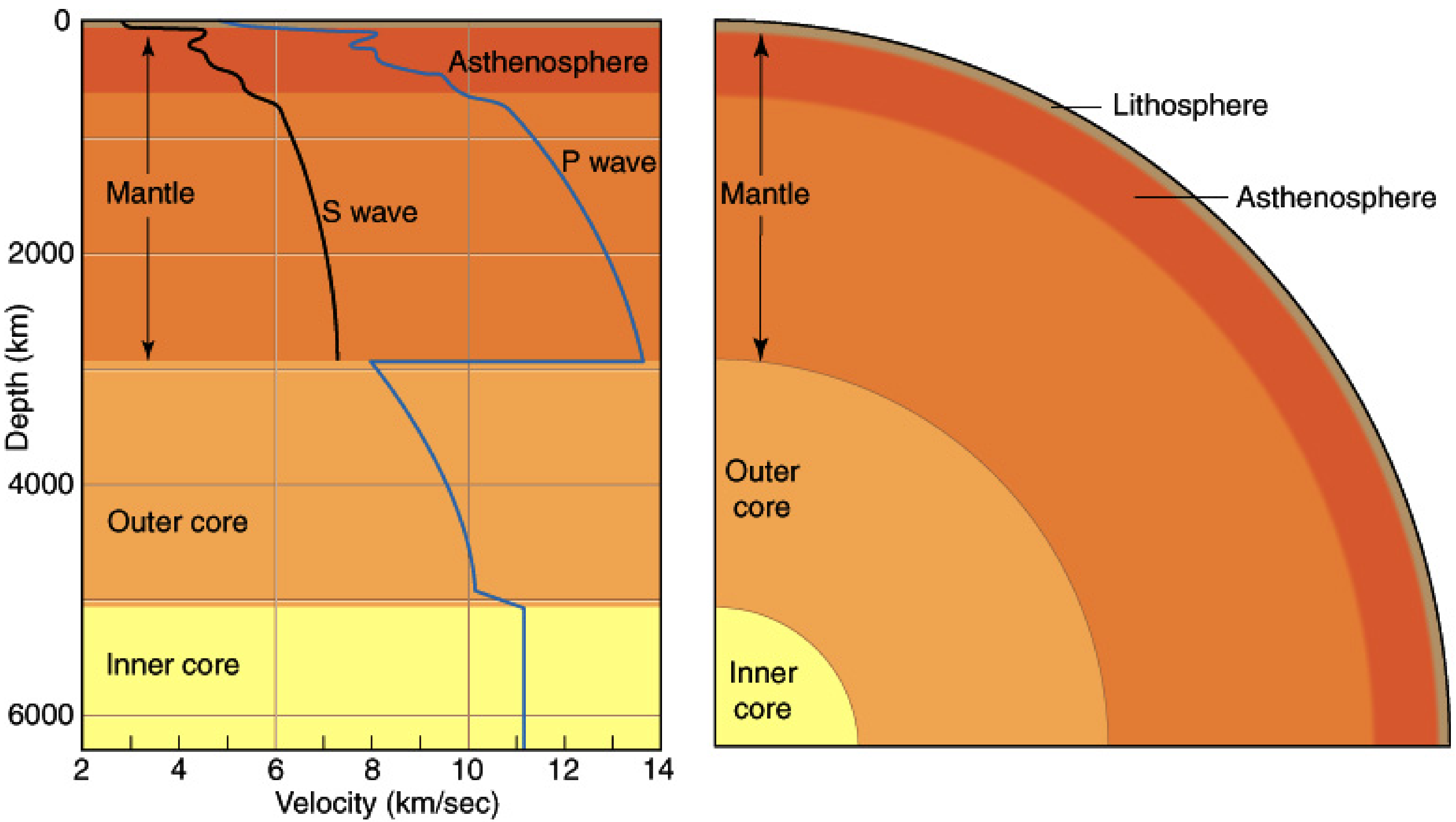
Travel-Time Curves

The sorted (by distance) ensemble of picked arrivals provide a travel time curve, which gives an "image of the wave propagation though the medium

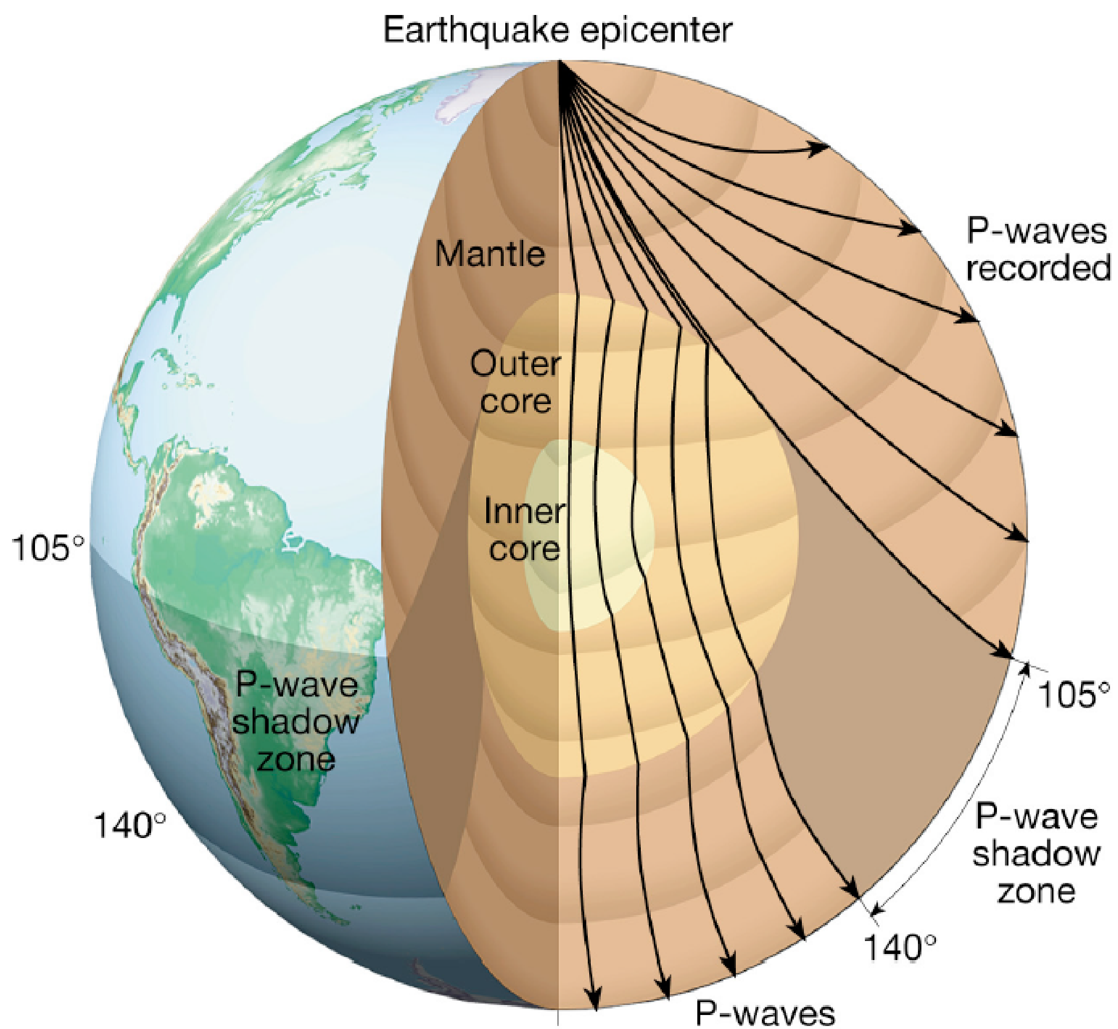


Earth Velocity Model

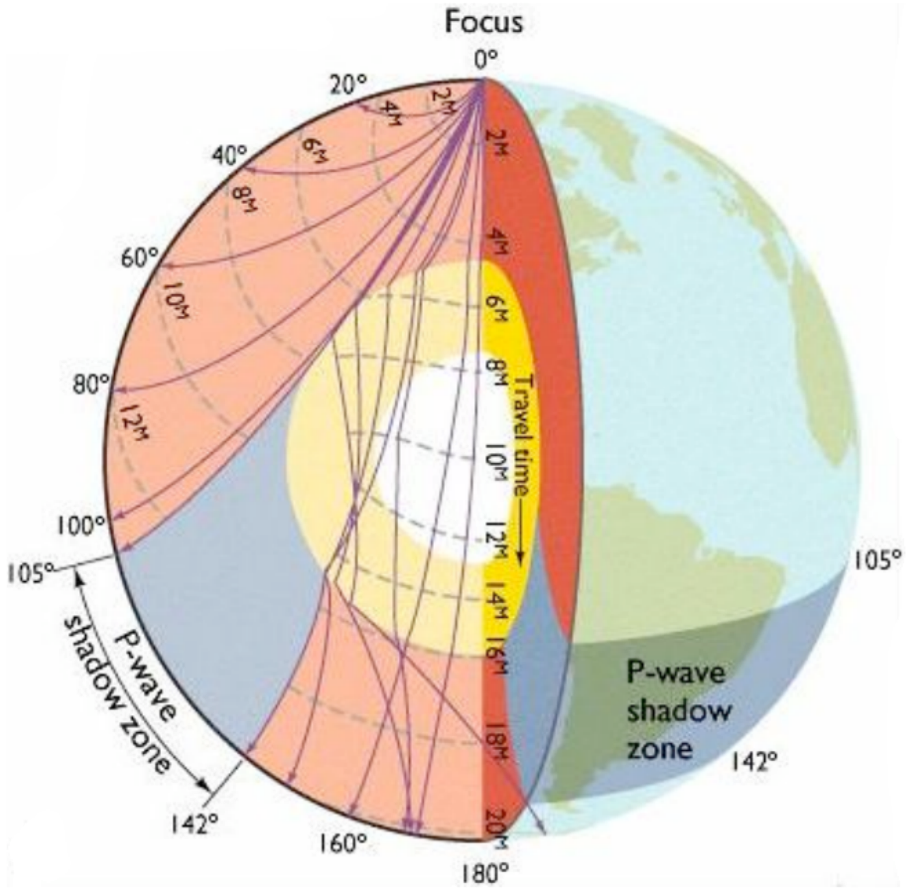
P wave velocities drop suddenly at 2900 km depth, and S waves cannot pass through this layer (the liquid outer core)

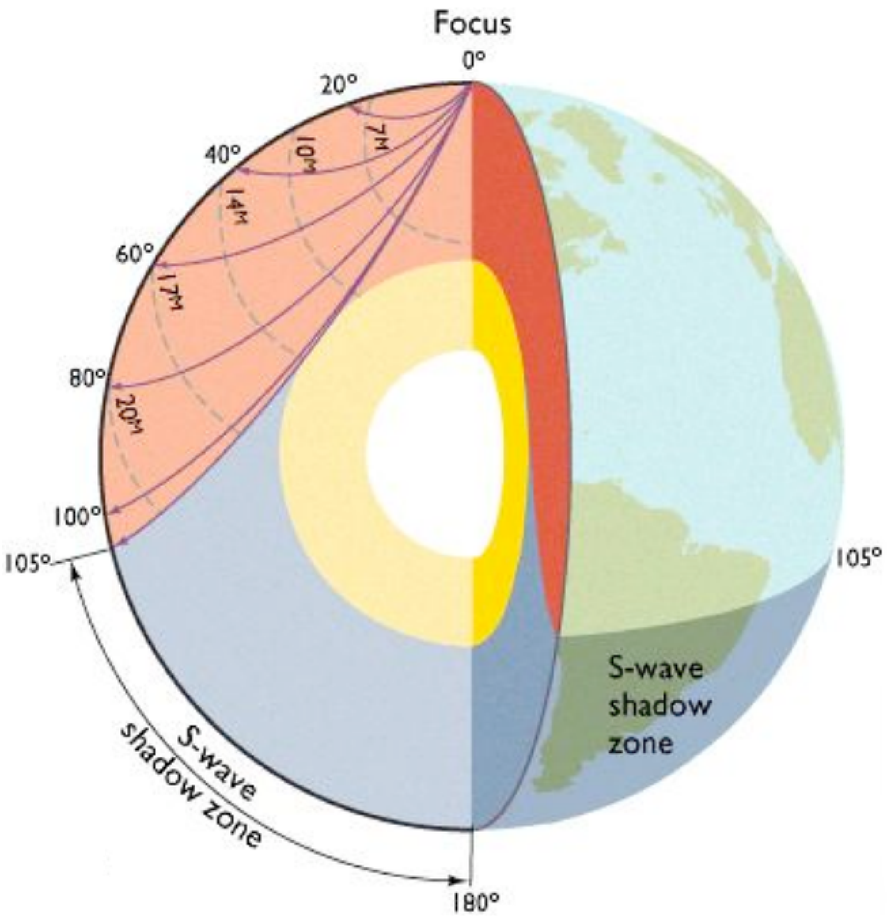


Waves Through Earth



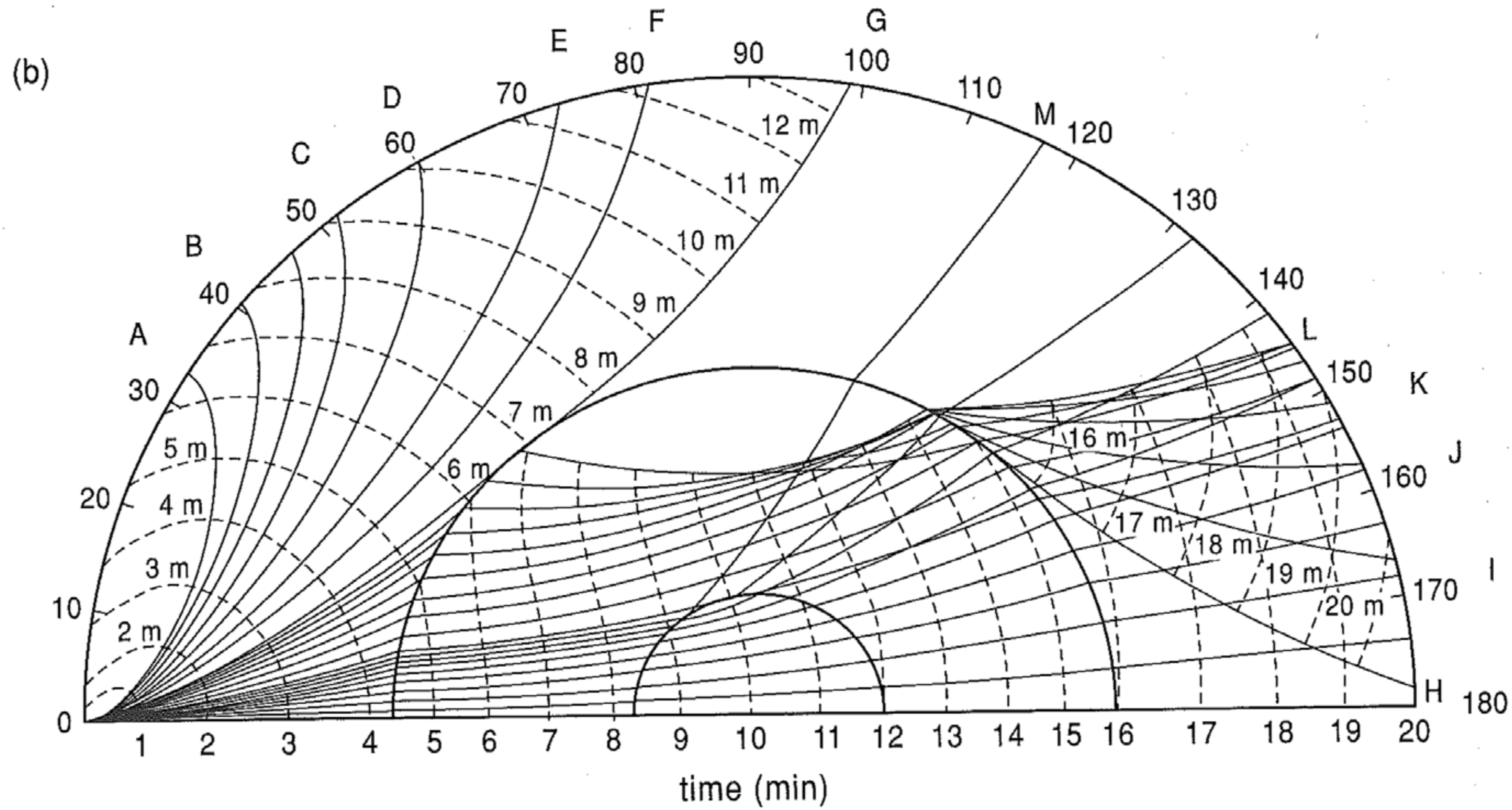
P and S Shadow Zones



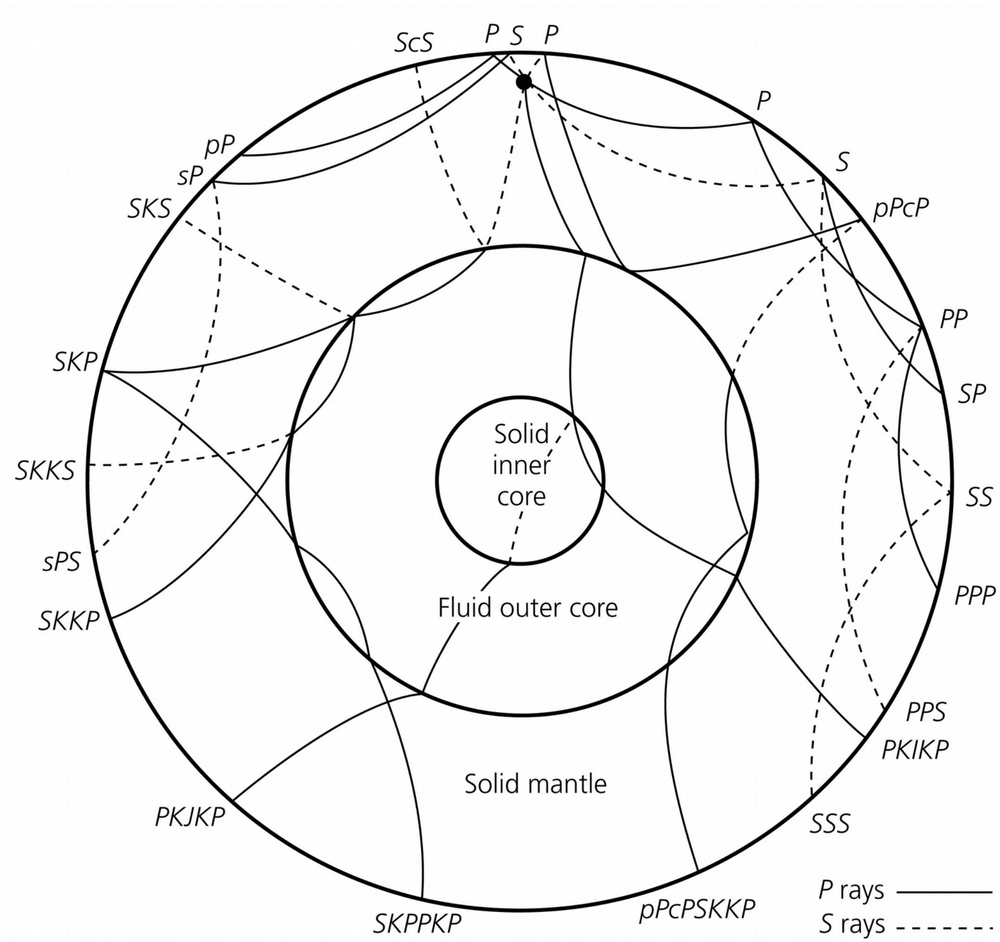


How long does it takes?

Seismic waves can travel through Earth in a matter of minutes!
Teleseismic: Rays that arrive at > 18deg away from their source → useful for investigating the deep Earth



Earth Phases

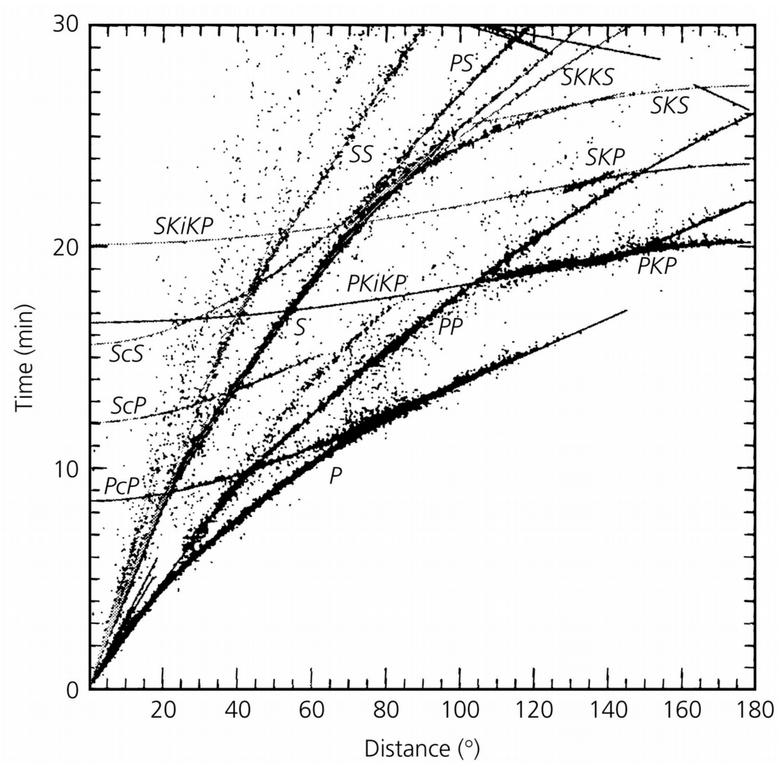


Naming conventions
(no need to memorize)

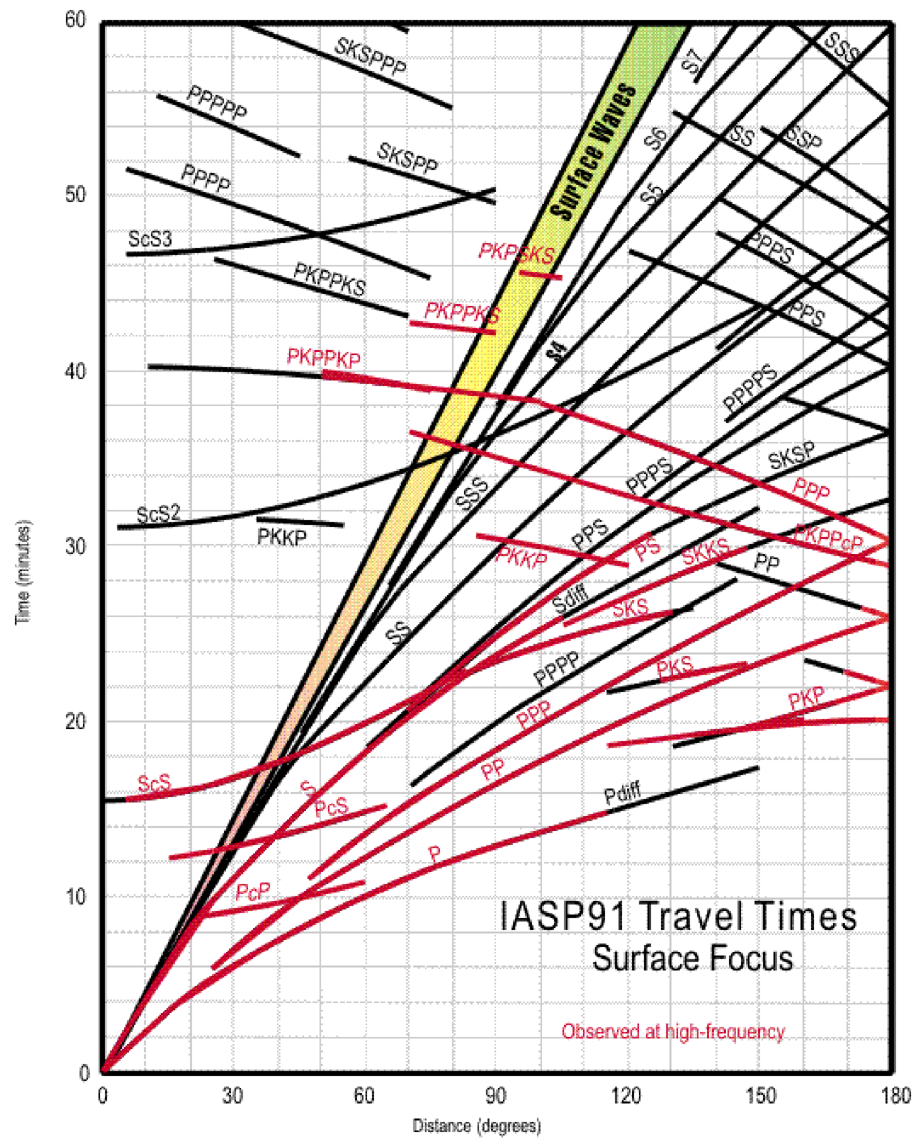
P: P–wave in the mantle
K: P–wave in the outer core
I: P–wave in the inner core
S: S–wave in the mantle
J: S–wave in the inner core
C: reflection off the core–mantle boundary (CMB)
I: reflection off the inner–core bounday (ICB)
PmP: reflection off of Moho
Pn: refracted wave on Moho
Pg: direct wave in the crust

Note: small and capitalized letters do matter!

Standard Travel Time Model



Matching observed arrival times with theoretical travel times we can find the distance at which an earthquake occurred



Other Examples: Crustal Phases