Engineering Seismology and Seismic Hazard – 2019

Lecture 9

Seismic Energy and Attenuation

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Kinetic Energy

Seismic waves transport both **kinetic and elastic strain** (or potential) energy as they propagate.

The kinetic energy in a volume V is the integral of the sum of the kinetic energy associated with each component of the displacement:

$$\boxed{E_k = \frac{1}{2} \int_V \rho \left(\frac{\partial u}{\partial t} \right)^2 dV}$$

Which is dimensionally analogous to the well known expression:

$$E_k = \frac{1}{2} m v^2$$

Kinetic Energy

For a plane wave, the volume integral reduces to a line integral. Assuming then a solution of the type:

$$u = A\cos(\omega t - kx)$$

the kinetic energy per unit area of wavefront averaged over a wavelength can therefore be written as:

$$E_k = \frac{1}{2\lambda}\int_0^{\lambda} \rho \left(\frac{\partial [A\cos(\omega t - kx)]}{\partial t} \right)^2 dx$$

$$\Rightarrow \quad E_k = \frac{A^2 \omega^2 \rho}{4}$$

Potential Energy

The strain or potential energy stored in a volume is simply the integral of the product of the stress and strain components summed together:

$$\boxed{E_P = \int_V \sigma_{ij}\, \varepsilon_{ij}\, dV}$$

As for the kinetic energy, by assuming a general solution it is:

$$\varepsilon = \frac{1}{2}\frac{\partial u}{\partial x} = \frac{1}{2} A k \sin(\omega t - k x)$$

$$\sigma = 2\mu\,\varepsilon = \mu A k \sin(\omega t - k x)$$

$$\Rightarrow \quad E_p = \frac{A^2 \omega^2 \rho}{4}$$

Total Energy

Total energy averaged over a wavelength is then the sum of kinetic energy and potential energy:

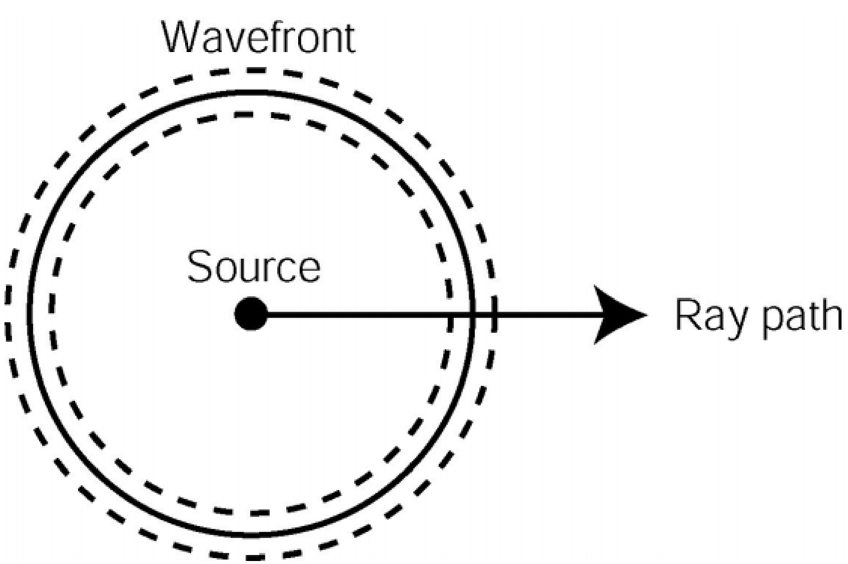
$$\boxed{E_{tot} = E_k + E_p = \frac{A^2 \omega^2 \rho}{2}}$$

The total energy is proportional to the square of the amplitude and the frequency. Thus, increasing the frequency with fixed amplitude will quadratically increase the energy carried by the wave, and vice versa.

Geometrical Spreading

Seismic energy spreads out from a point source (e.g., explosion) as a spherical wavefront. The wavefront always contains a constant amount of energy.

At each time T, energy at a point on the sphere surface is:



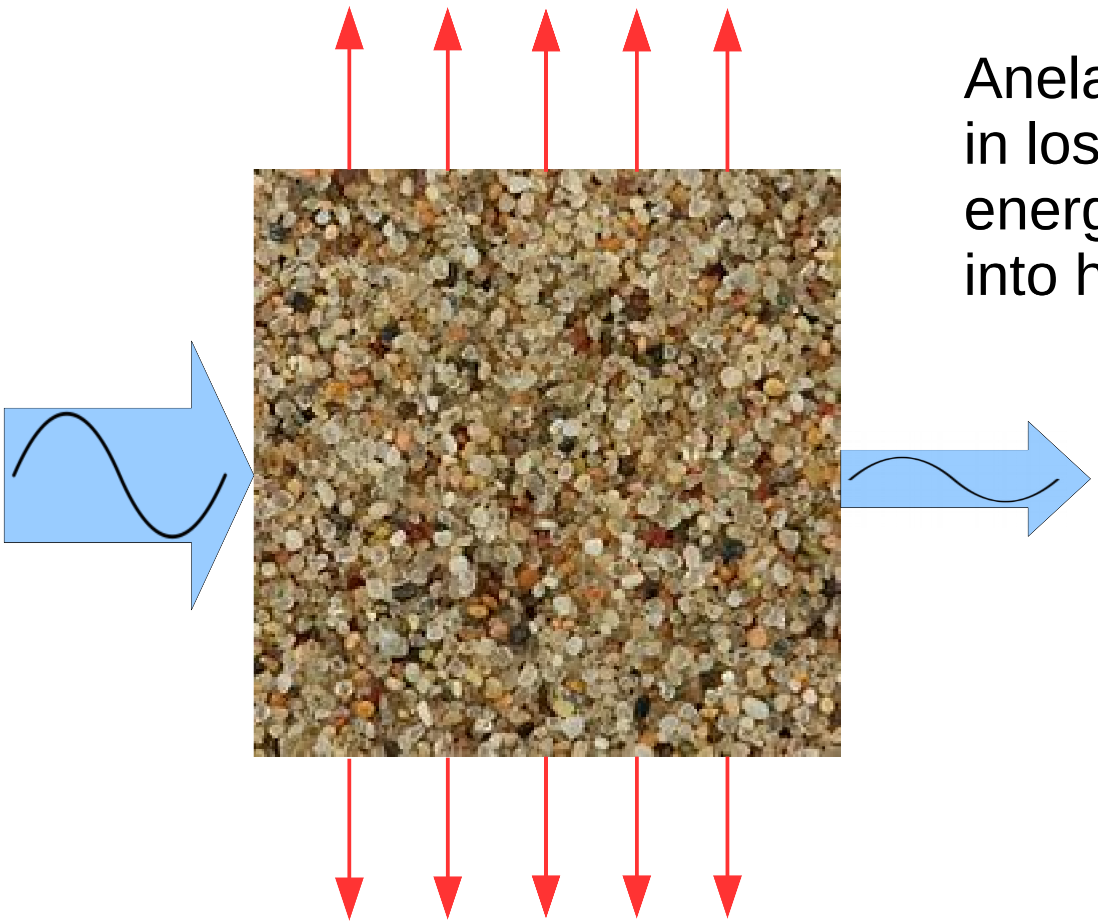
$$E_s = \frac{E_{tot}}{4\pi r^2} = \frac{A^2 \omega^2 \rho}{\pi r^2}$$

Since energy is proportional to the square of amplitude, amplitude will decay proportionally to 1/r

$$Es \propto 1/r^2 \Rightarrow A \propto r$$

Anelastic Attenuation

Wave amplitudes is also reduced because of energy loss due to inelastic material behavior or internal friction during wave propagation. These effects are called anelastic of intrinsic attenuation.



Anelastc attenuation results in loss of mechanical energy, by transformation into heat

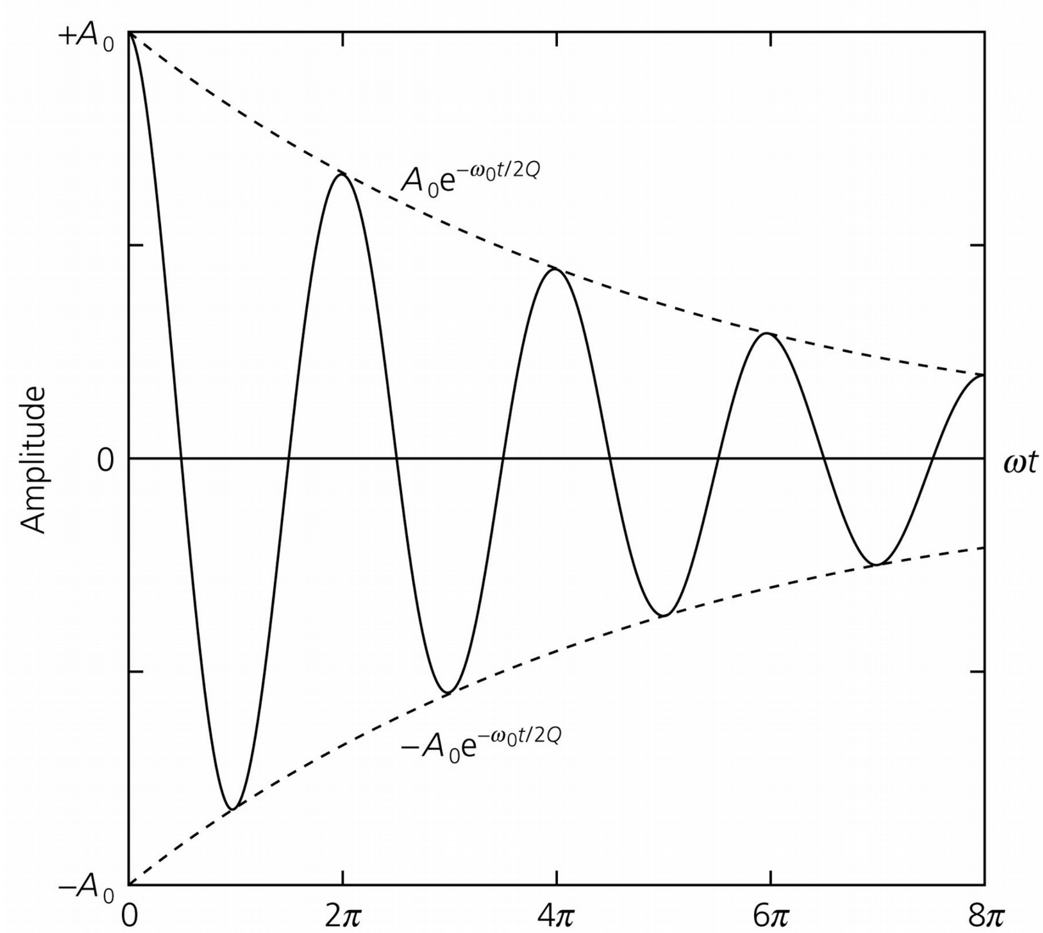
Quality Factors

The wave attenuation is usually expressed in terms of the dimensionless quality factor Q, which quantifies the energy loss per cycle.

$$Q = 2\pi \left(\frac{E}{\delta E} \right)$$

The amplitude decay due to energy dissipation can be written:

$$\boxed{A = A_0 e^{-\frac{\omega t}{2Q}} = A_0 e^{-\frac{\omega x}{2Qv}}}$$



Complex Velocities

The harmonic wave solution can be written to account for anelastic attenuation:

$$A = e^{i(\omega t - kx)} e^{-\frac{\omega x}{2Qv}} = e^{i\omega\left(t - \frac{x}{v}\right)} e^{-\frac{\omega x}{2Qv}} = e^{i\omega\left(t - \frac{x}{v}\left(1 - \frac{1}{2Qi}\right)\right)}$$

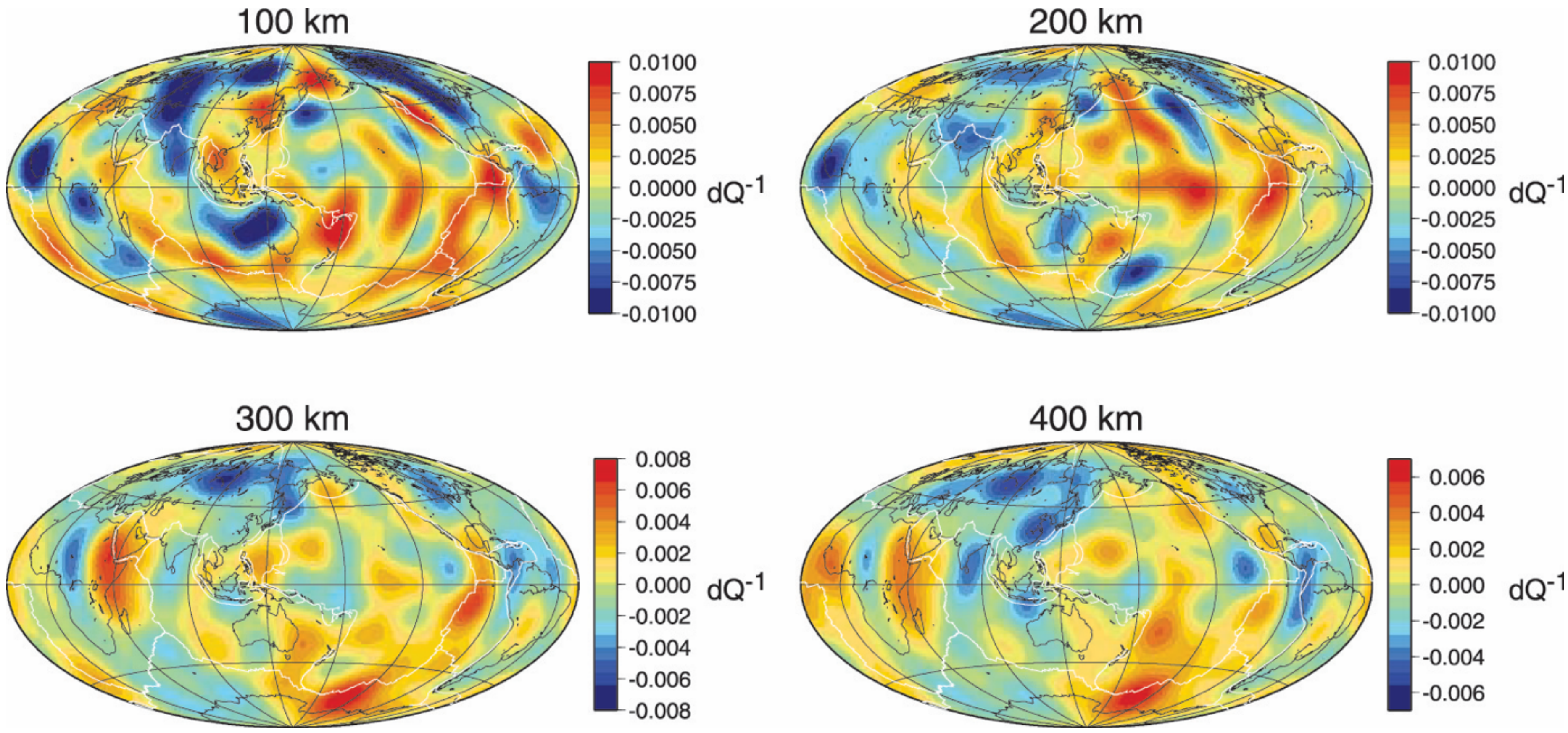
By substitution of a generic complex velocity term we get:

$$\boxed{v^{*} = v\left(\frac{2Qi}{2Qi - 1}\right)} \Rightarrow A = e^{i\omega\left(t - \frac{x}{v^{*}}\right)}$$

In this way, all mathematical formulations valid for the elastic case can be used for the anelastic.

Global Attenuation Models

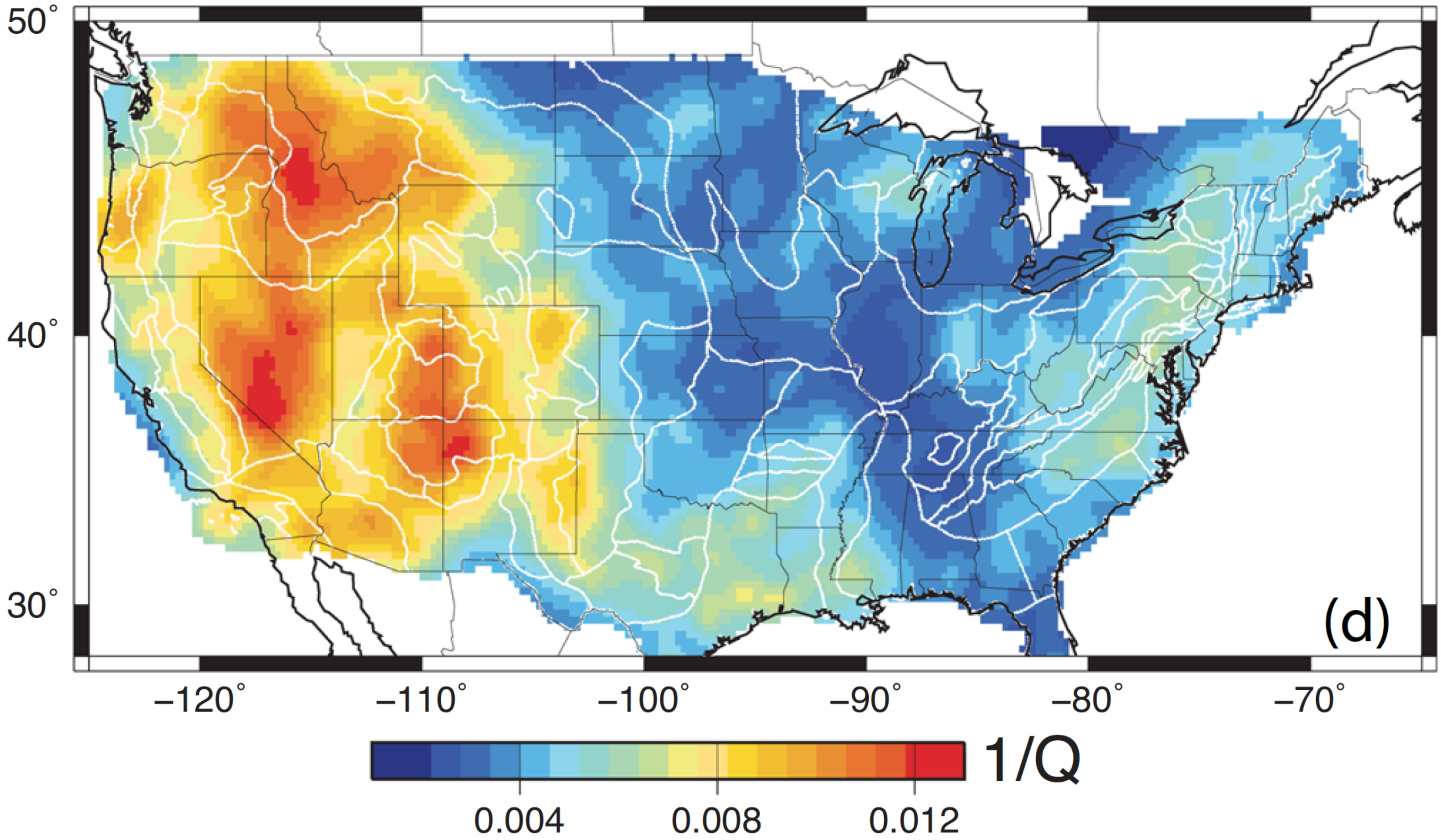
Attenuation models can be obtained by seismic tomography and surface wave analysis.



Dalton et al. 2008

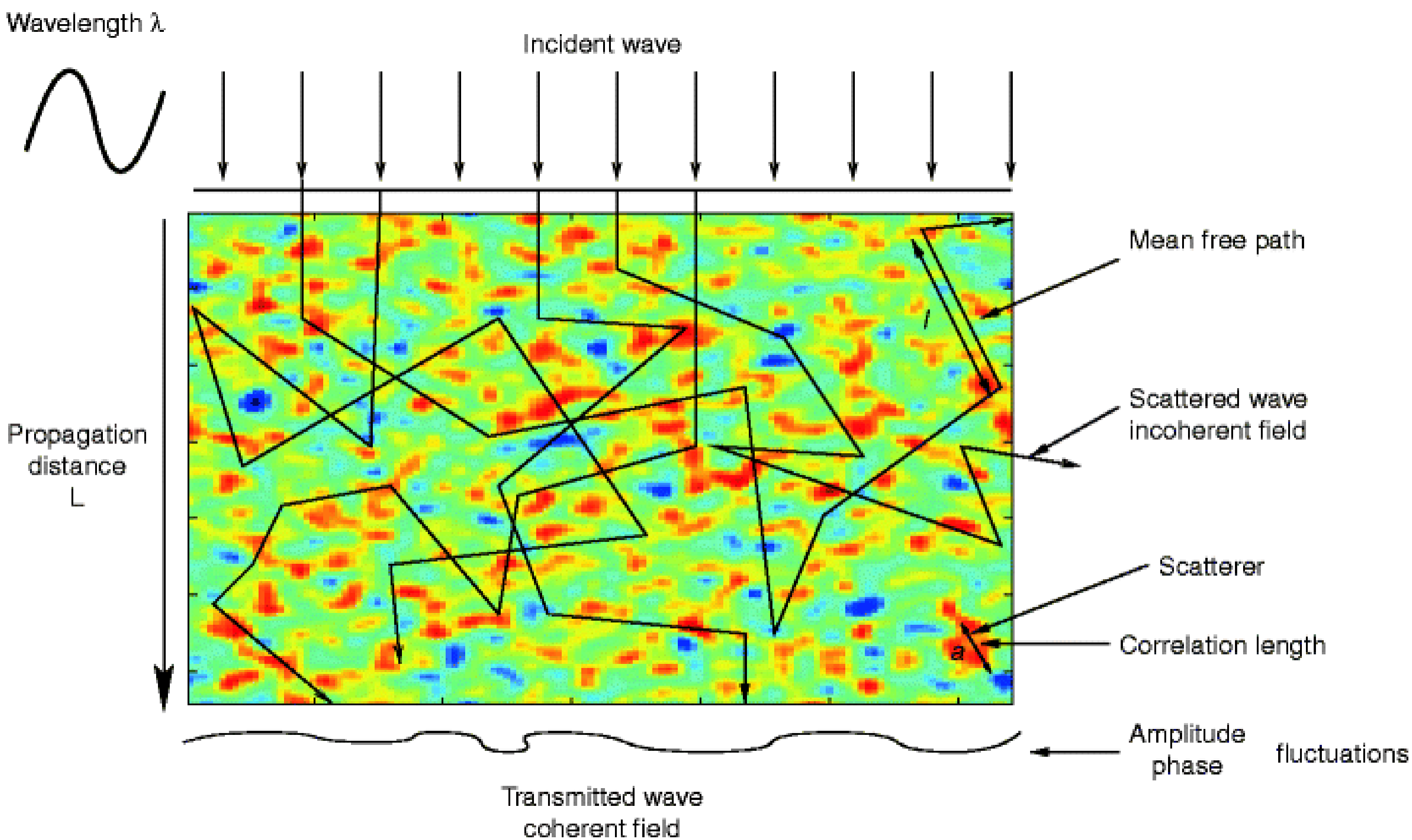
Relevance for Seismic Hazard

Regionalization of attenuation is fundamental in ground motion prediction for seismic hazard analysis. Lower attenuation implies propagation over larger distances.



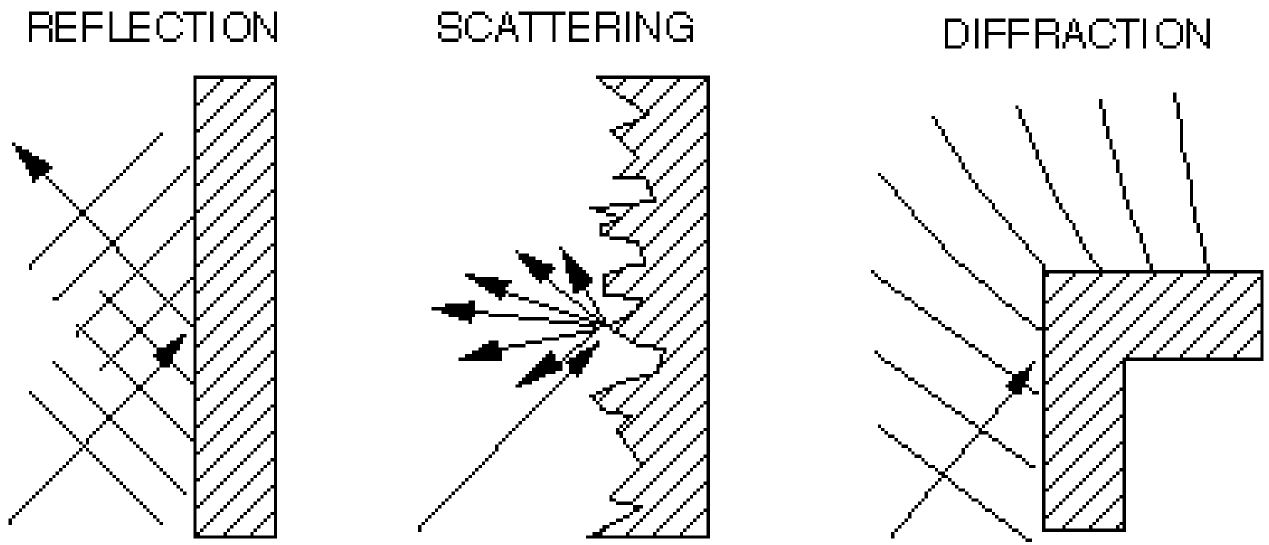
Scattering

Scattering attenuation is due to the inhomogeneous structure of the earth at much smaller scale than the wavelength of interest.



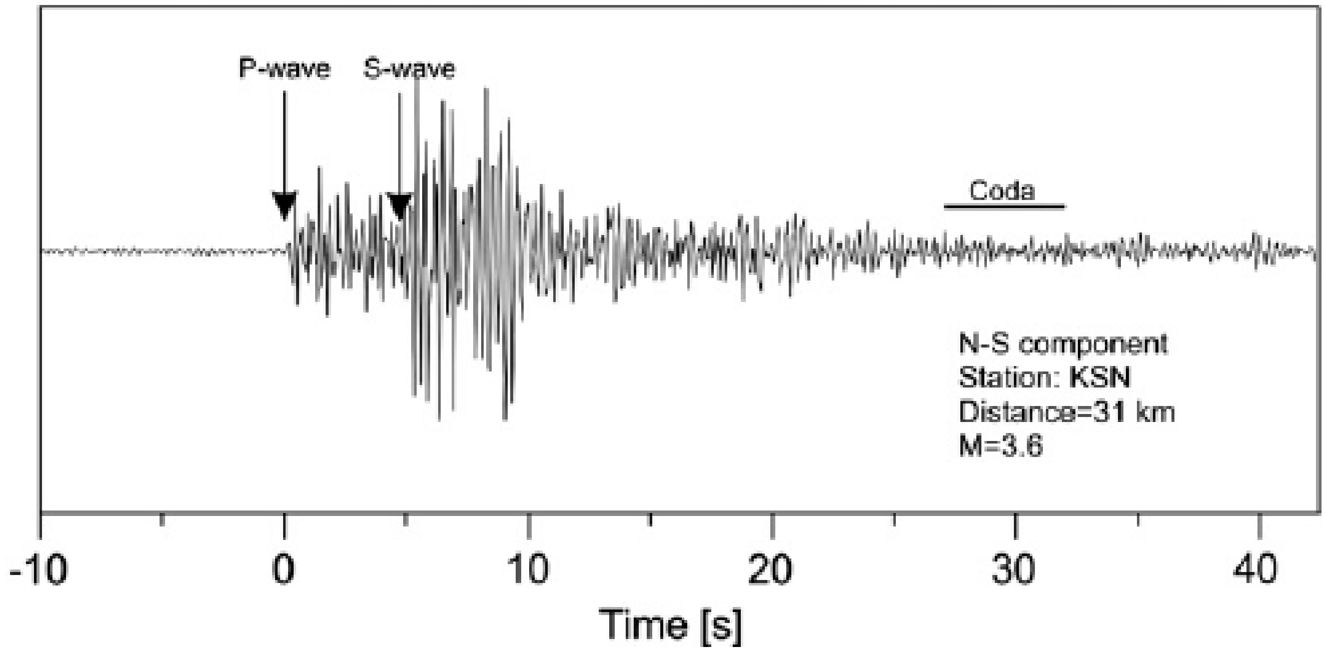
Scattering

Scattering does no only affects propagation through material, but also reflection and transmission trough irregular interfaces.



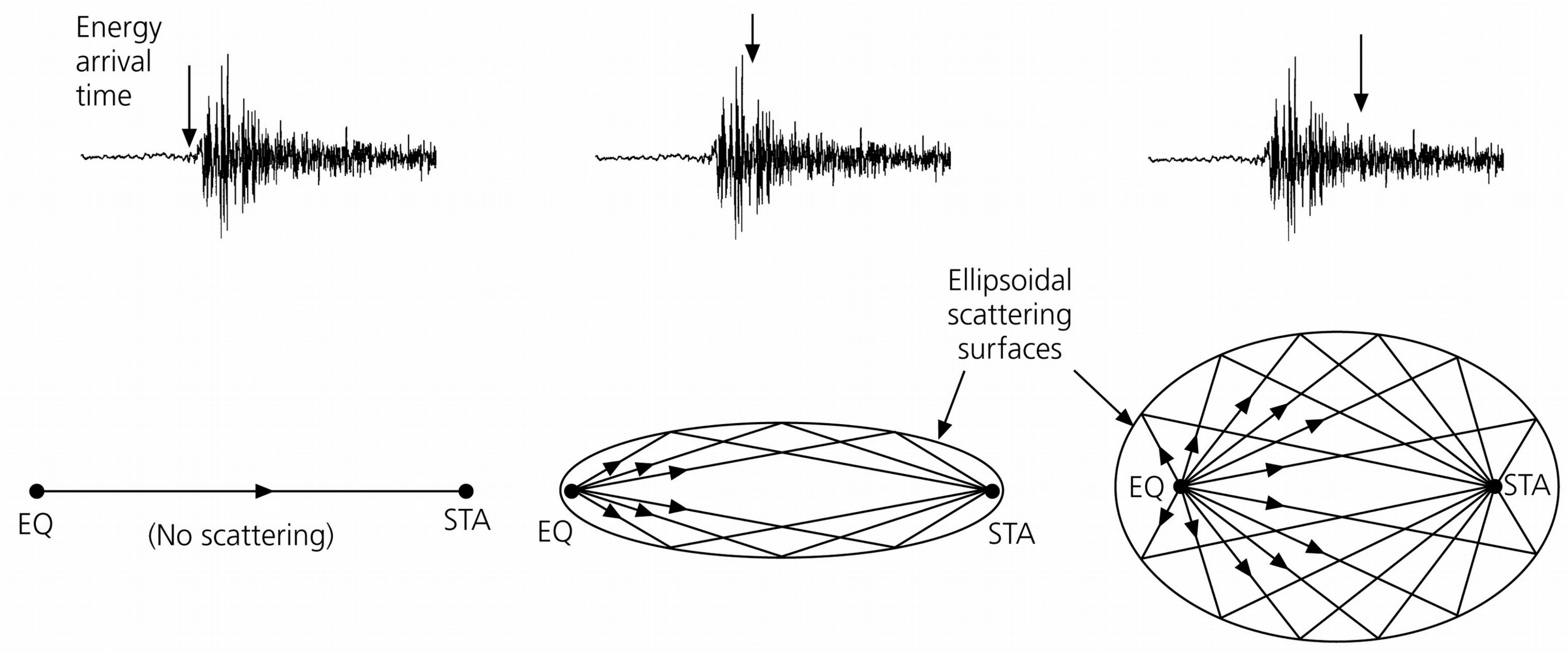
Coda Waves

The most visible result of scattering is the creation of long tail of reverberating waves (coda) after the main seismic phases.



While amplitude of first arrivals decreases, total seismic energy is distributed over a window of longer duration (can overlap surface wave arrivals).

Geometric interpretation



Moonquakes Case

Moonquakes are very different to earthquakes, mostly due to the large scatter induce by the Moon's regolith, a layer of loose, highly heterogeneous superficial deposits covering the solid bed rock.