Engineering Seismology and Seismic Hazard – 2019

Lecture 10

The Seismic Source

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Harmonic Solution Recap

So far, we have considered the dynamic case where no body force were acting on the medium:

$$(\lambda + 2\mu)\nabla(\nabla \cdot \vec{u}) - \mu \nabla \times \nabla \times \vec{u} = \rho \frac{\partial^2 \vec{u}}{\partial t^2}$$

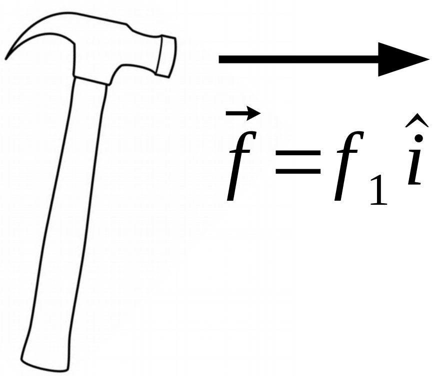
Wave equation solution was obtained by solving Helmot'z potentials, which lead to a purely harmonic solution:

$$\vec{u}(\vec{x},t) = \left(A \nabla e^{-i \vec{\kappa}_\alpha \cdot \vec{x}} + B \nabla \times \hat{n} e^{-i \vec{\kappa}_\beta \cdot \vec{x}} \right) e^{i(\omega t)}$$

External Point Force

We now add the action of an external point force, such as a Dirac pulse:

$$(\lambda+2\mu)\nabla(\nabla\cdot\vec{u})-\mu\nabla\times\nabla\times\vec{u}+\vec{f}=\rho\frac{\partial^2\vec{u}}{\partial t^2}$$



$$\vec{f}=f_1\hat{i}+f_2\hat{j}+f_3\hat{k}$$

It looks a rather simple case.....

Point Source Solution

Without entering into tedious mathematics demonstration, the solution will be:

$$u_i(\vec{x},t) = \frac{1}{4\pi\rho}(3\gamma_i\gamma_j - \delta_{ij})\frac{1}{r^3}\int_{r/\alpha}^{r/\beta}\tau f_j(t-\tau)\,d\tau +$$

$$\frac{1}{4\pi\rho\alpha^2}\gamma_i\gamma_j\frac{1}{r}f_j\left(t-\frac{r}{\alpha}\right) +$$

$$\frac{1}{4\pi\rho\beta^2}(\delta_{ij}-\gamma_i\gamma_j)\frac{1}{r}f_j\left(t-\frac{r}{\beta}\right)$$

Near field

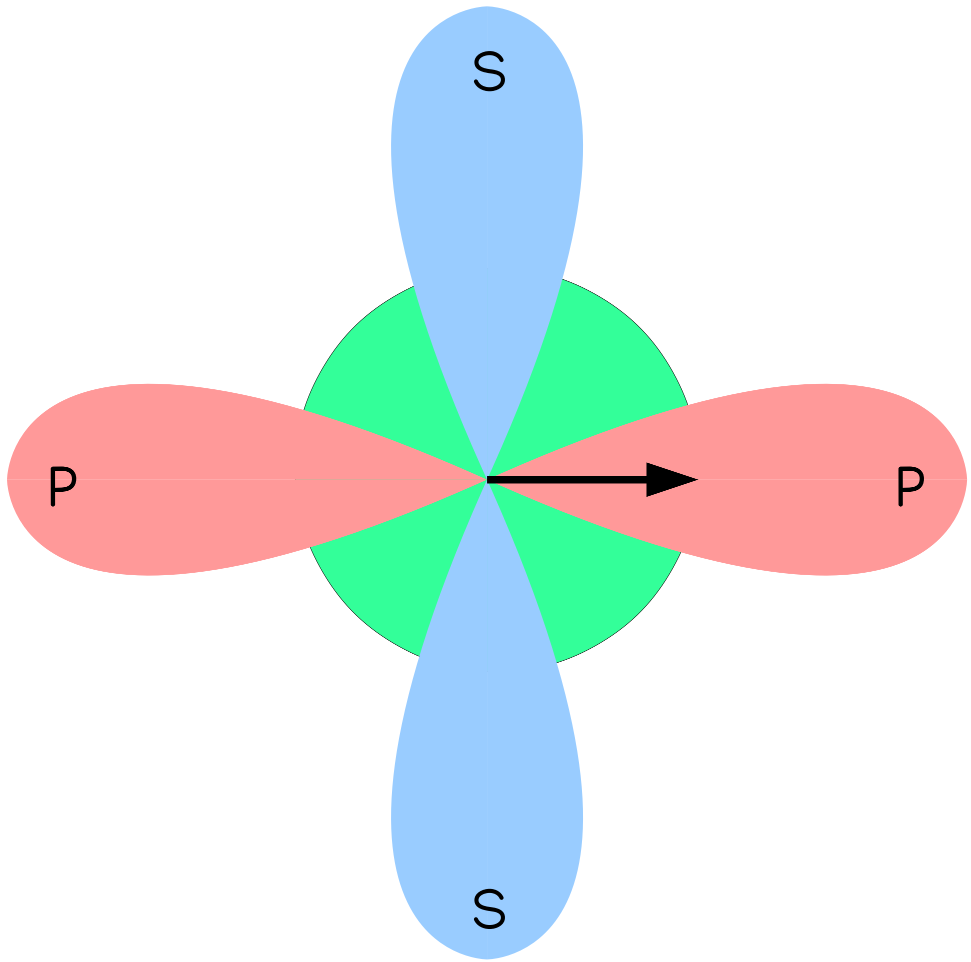
P-wave far field

S-wave far field

Where γ_i and γ_j are the direction cosines

Radiation Pattern of a Point Source

The P and S wave radiation patter is highly directional and depends on the relative location and distance to the observer.



Green's Tensor Convolution

The solution can also be expressed by convolution of a tensor containing the information about the medium and the point source

$$\boxed{u_i(\vec{x},t)=G_{ij}(\vec{x},t,\vec{x}_0,t_0)f_j(\vec{x}_0,t_0)}$$

Where G is the **Green's Tensor**:

$$G_{ij}(\vec{x}_0,t_0)=\frac{1}{4\pi\rho}(3\gamma_i\gamma_j-\delta_{ij})\frac{1}{r^3}t\left[H\left(t-\frac{r}{\alpha}\right)-H\left(t-\frac{r}{\beta}\right)\right]+$$

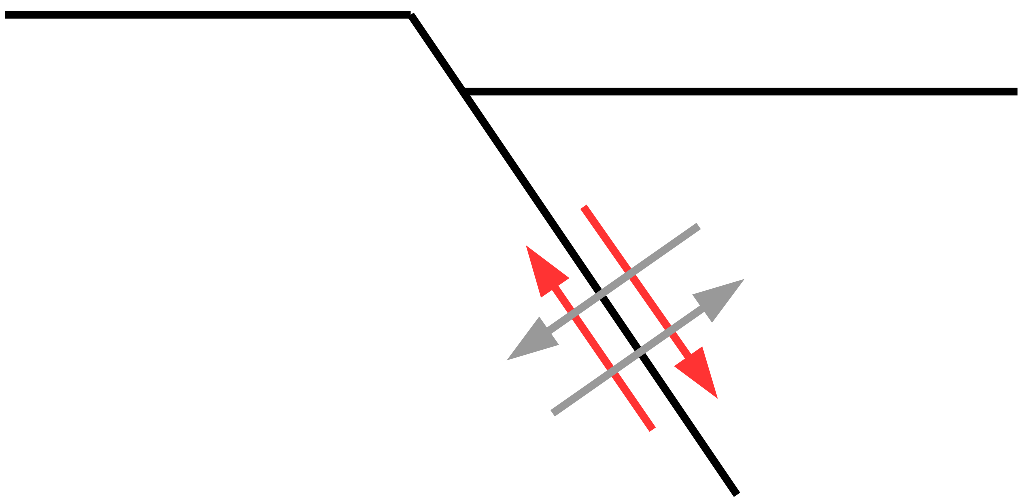
$$\frac{1}{4\pi\rho\alpha^2}\gamma_i\gamma_j\frac{1}{r}\delta\left(t-\frac{r}{\alpha}\right)+$$

$$\frac{1}{4\pi\rho\beta^2}(\delta_{ij}-\gamma_i\gamma_j)\frac{1}{r}\delta\left(t-\frac{r}{\beta}\right)$$

A Realistic Source Representation

We know that earthquakes are generated by faults and a point load is not a good approximation.

In order to represent the dynamic of a slipping fault we need to use a force couple.

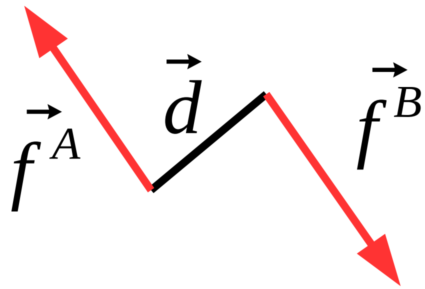


<u>Actually, we need two counteracting force couples to avoid rotation (torque) of the system (faults do not rotate!)</u>

Force and Moment

Each force acting in the couple can be represented though its moment:

$$M_{jk} = f_j^A d_k$$



The dynamic displacements can than be represented by Taylor's expansion using the Greens' tensor as:

$$u_i = u_i^B + u_i^A = G_{ij}(0) f_j - G_{ij}(d_k) f_j = \frac{\delta G_{ij}}{\delta x_k} f_j d_k$$

$$\boxed{u_i = \frac{\delta G_{ij}}{\delta x_k} M_{jk}}$$

Force Couple Solution

$$u_i(\vec{x},t)=\frac{1}{4\pi\rho}R_{ijk}^{N}\frac{1}{r^4}\int_{r/\alpha}^{r/\beta}\tau M_{jk}(t-\tau)\,d\tau+$$

Near field

$$\frac{1}{4\pi\rho\alpha^2}R_{ijk}^{IP}\frac{1}{r^2}M_{jk}\left(t-\frac{r}{\alpha}\right)+$$

P wave middle field

$$\frac{1}{4\pi\rho\beta^2}R_{ijk}^{IS}\frac{1}{r^2}M_{jk}\left(t-\frac{r}{\beta}\right)+$$

S wave middle field

$$\frac{1}{4\pi\rho\alpha^3}R_{ijk}^{FP}\frac{1}{r}\frac{\delta M_{jk}}{\delta t}\left(t-\frac{r}{\alpha}\right)+$$

P wave far field

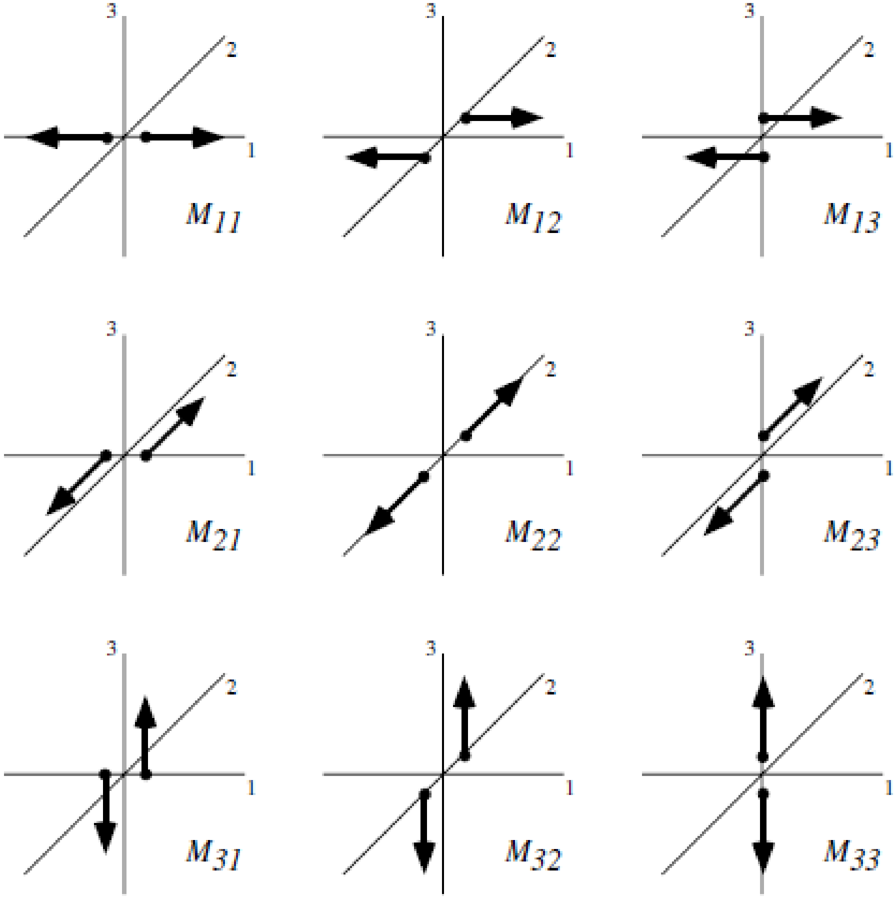
$$\frac{1}{4\pi\rho\beta^3}R_{ijk}^{FS}\frac{1}{r}\frac{\delta M_{jk}}{\delta t}\left(t-\frac{r}{\beta}\right)$$

S wave far field

The Moment Tensor

Each elements of the moment tensor represents a possible orientation of a force couple in space (on a given reference system)

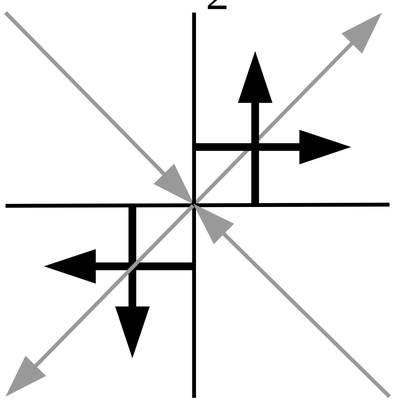
$$M_{jk} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

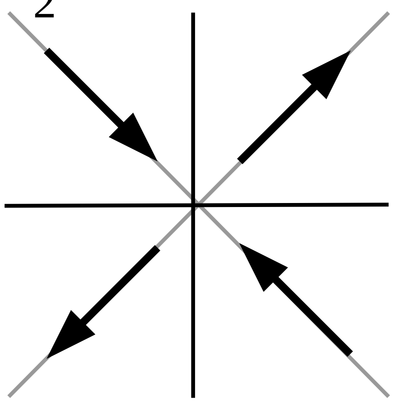


P-T Axes

The moment tensor is symmetric and it can be diagonalized.

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \qquad M' = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix}$$





<u>The double coupe is expressed by only two elements of M</u>

Scalar Seismic Moment

In case of shear dislocation (as a fault), since the total moment is zero, the moment tensor have null trace, one of the eigenvalues is 0 while the other two are equal.

$$M = \frac{M_0}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow M' = \frac{M_0}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

M_0 is the **Scalar Seismic Moment**, a scalar quantity related to the area of the fault and to the slip (averaged over the fault):

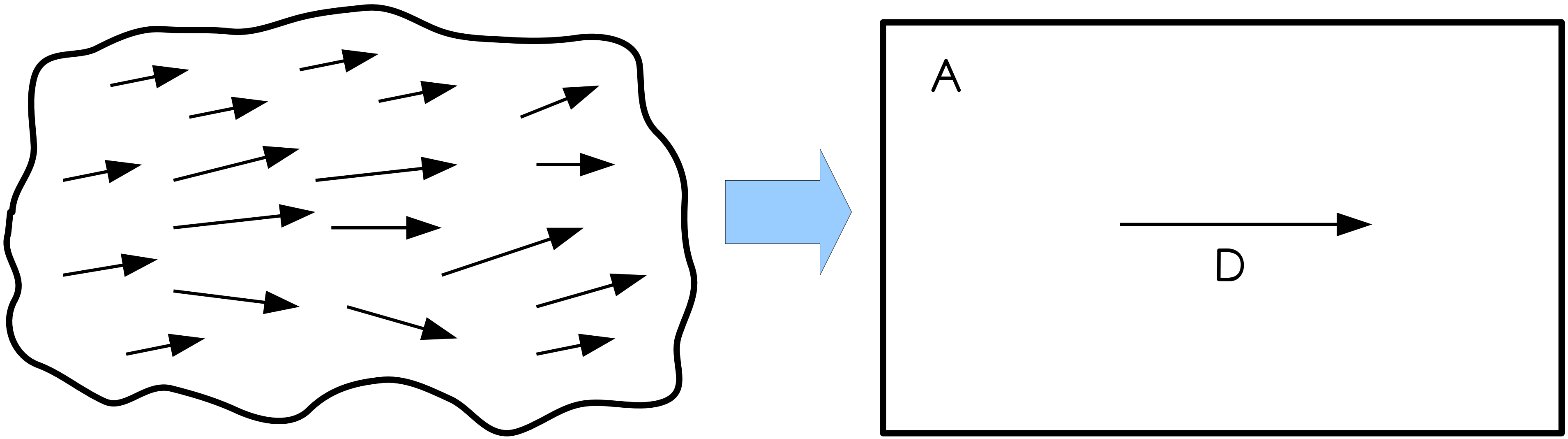
$$M_0 = \frac{1}{\sqrt{2}} \left(\sum_{ij} M_{ij}^2 \right)^{1/2}$$

Scalar Seismic Moment

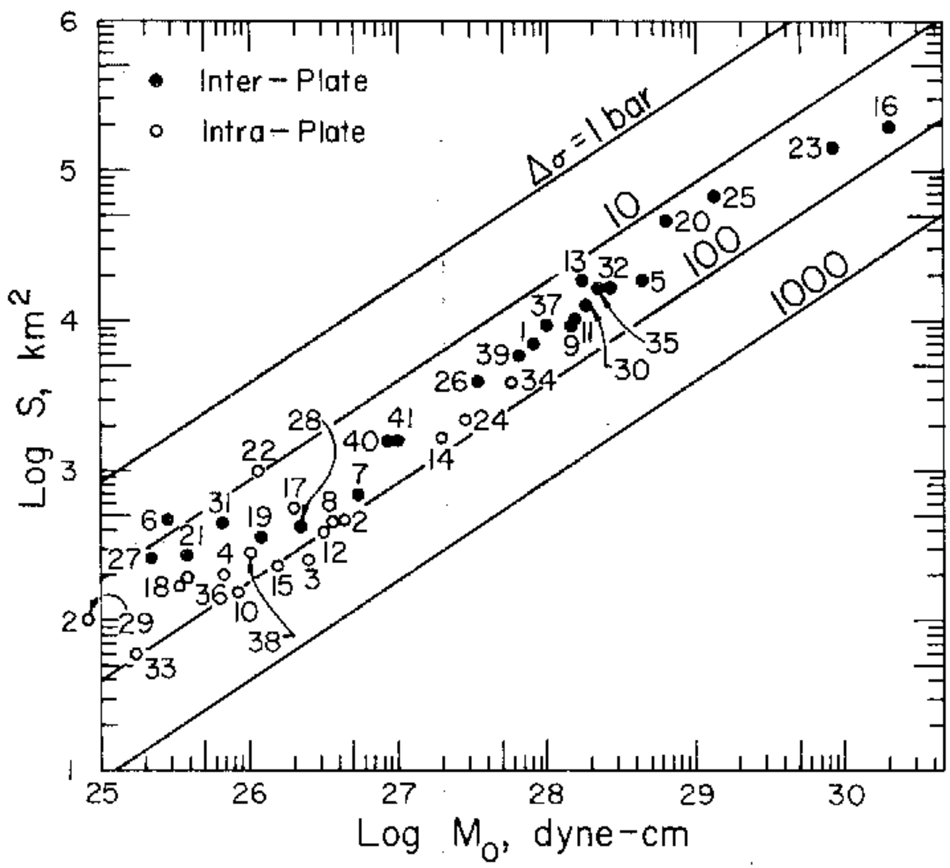
Seismic moment is a useful way to measure the size of an earthquake. Far a fault, it can be quantified as:

$$\boxed{M_0 = \mu\, A\, D = \mu\, L\, W\, D}$$

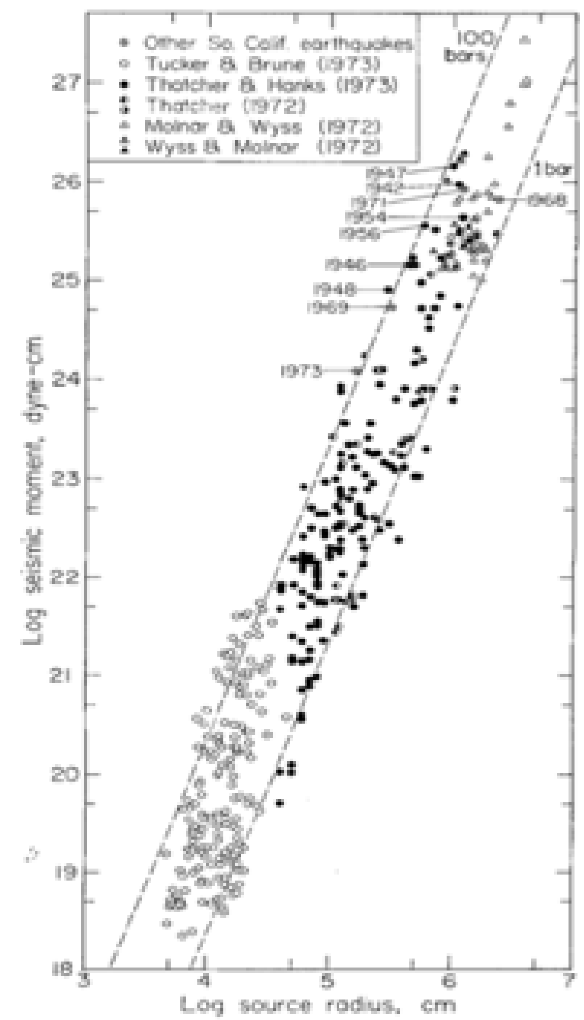
where μ is the shear modulus (rigidity), A is the fault area and D is the (average) dynamic slip during the earthquake.



Empirical Evidence: Scaling Laws

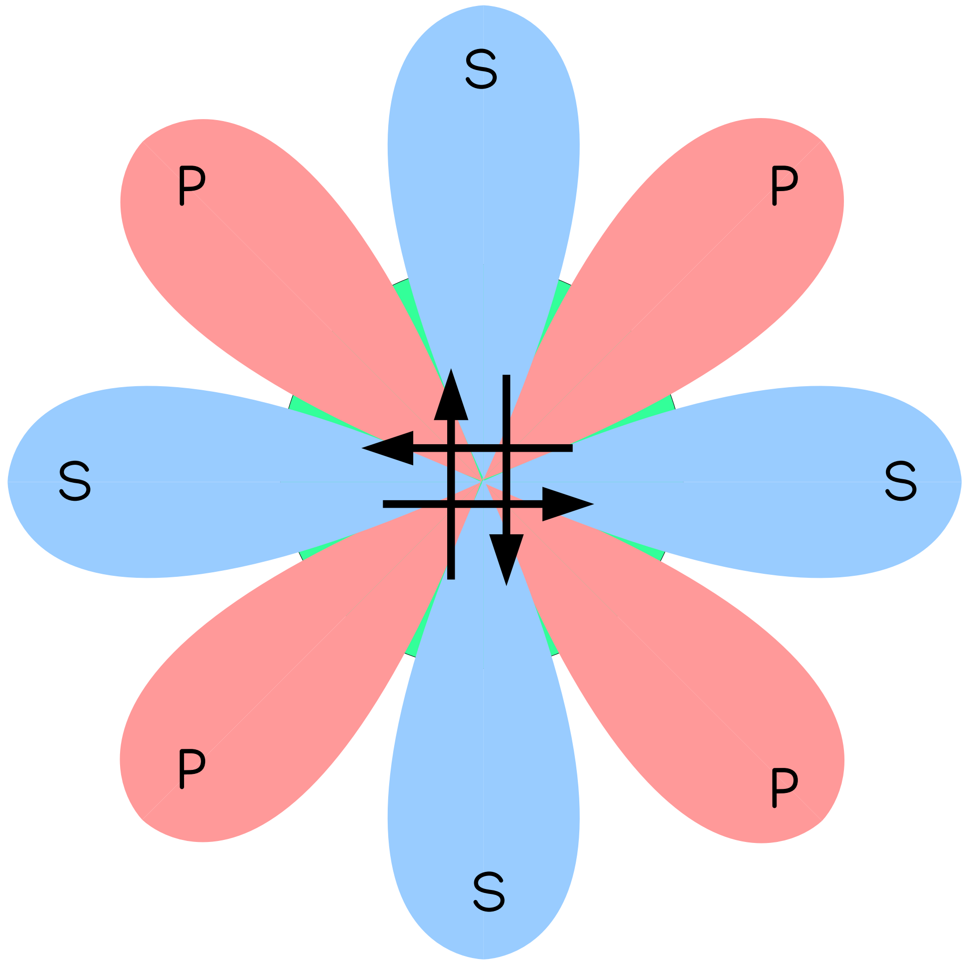


From Kanamori and Anderson 1975



From Hanks 1977

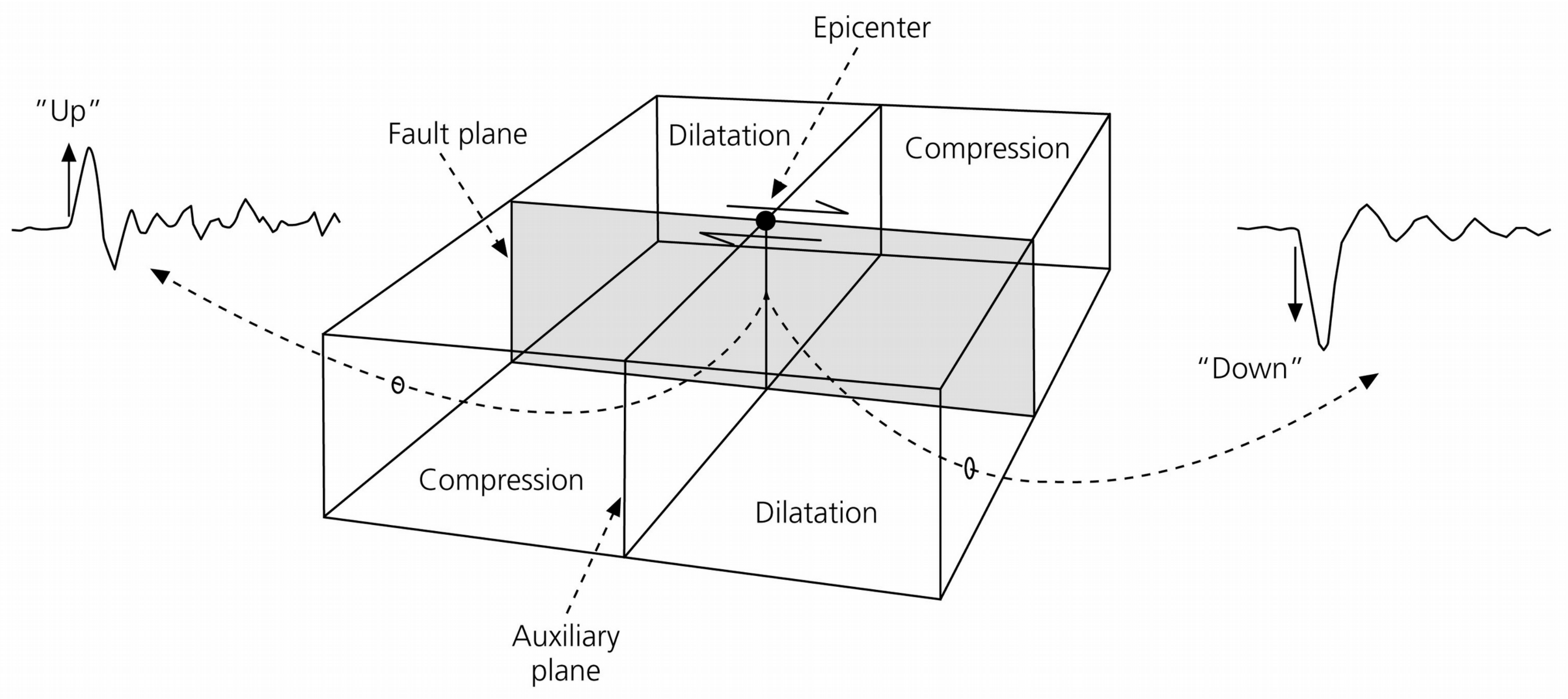
Radiation Pattern of a Double Couple



Where is the fault plane? Vertical or horizontal?

Facal Mechanism Solution

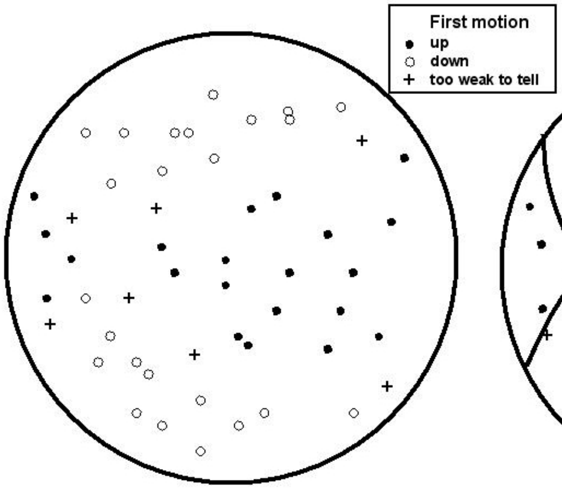
Due to radiation pattern, first arrivals have different polarity depending on the relative orientation of the recording station to the fault.



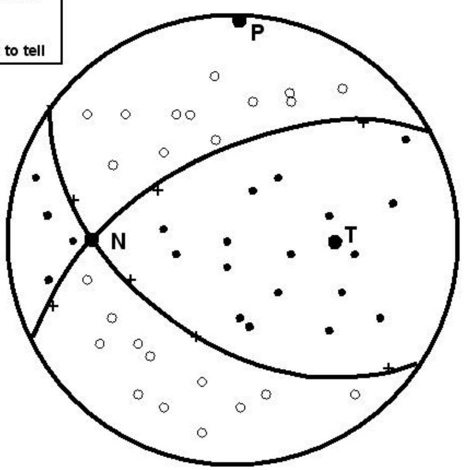
Wave polarity are analyzed to get fault orientation, although ambiguity between fault and auxiliary plane cannot be solved.

Beach Balls

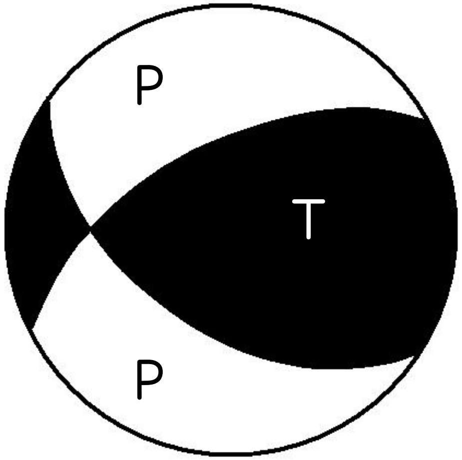
We use the stereographic projection to plot the polarity from various stations. Fault and auxiliary plane are then located.



First motion data for a hypothetical earthquake from various seismograph stations



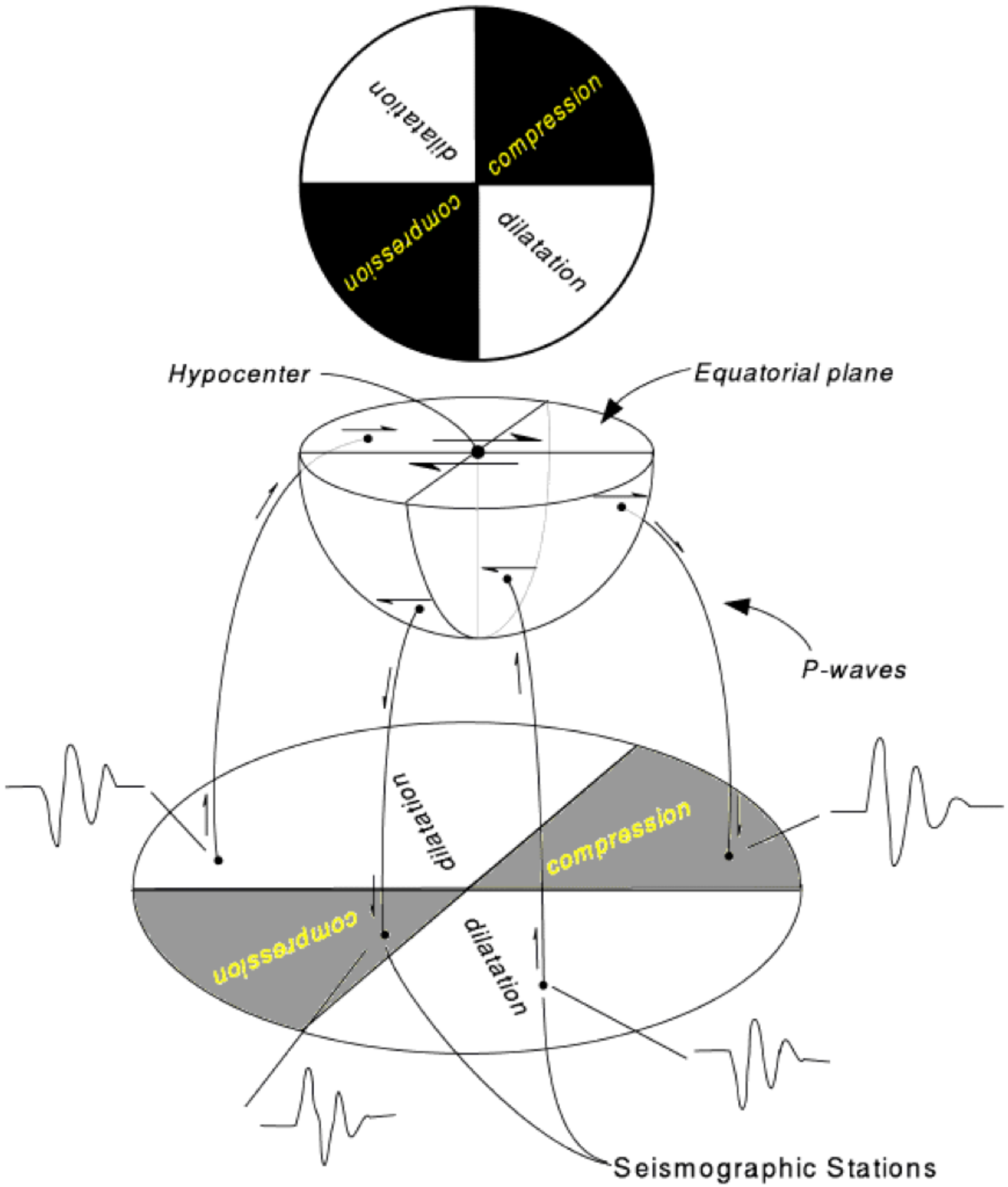
Nodal planes and N, P & T axes fitted to the data



Resultant ‘beachball’ plot showing that the earthquake resulted from reverse oblique movement on a fault of one of two possible orientations

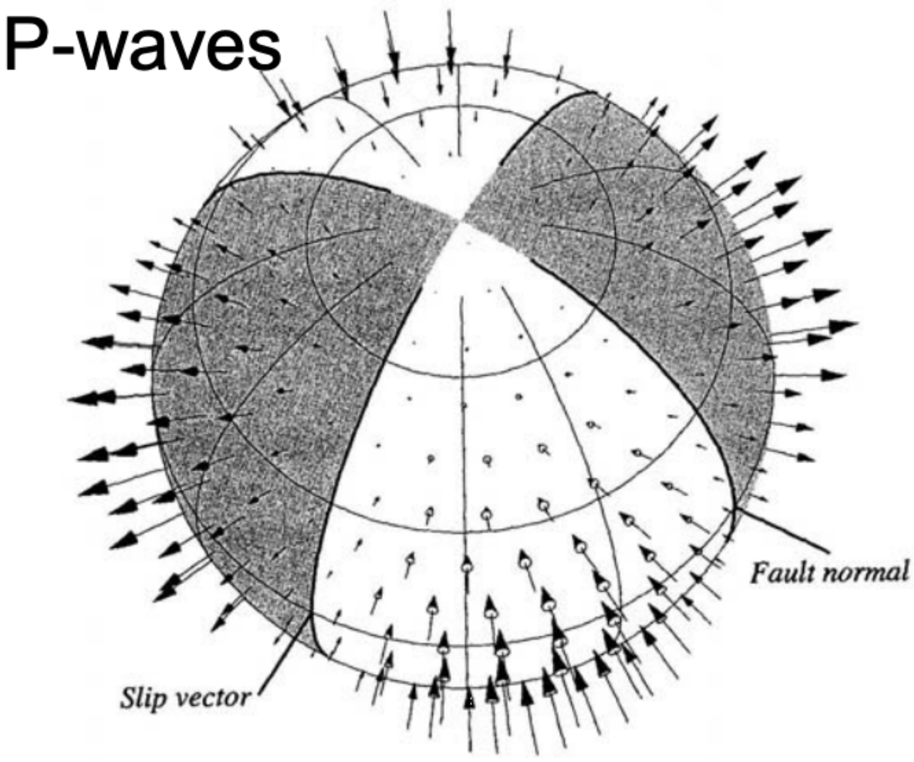
White conventionally indicates the axis of maximum pressure (P), while black the axis of maximum tension (T)

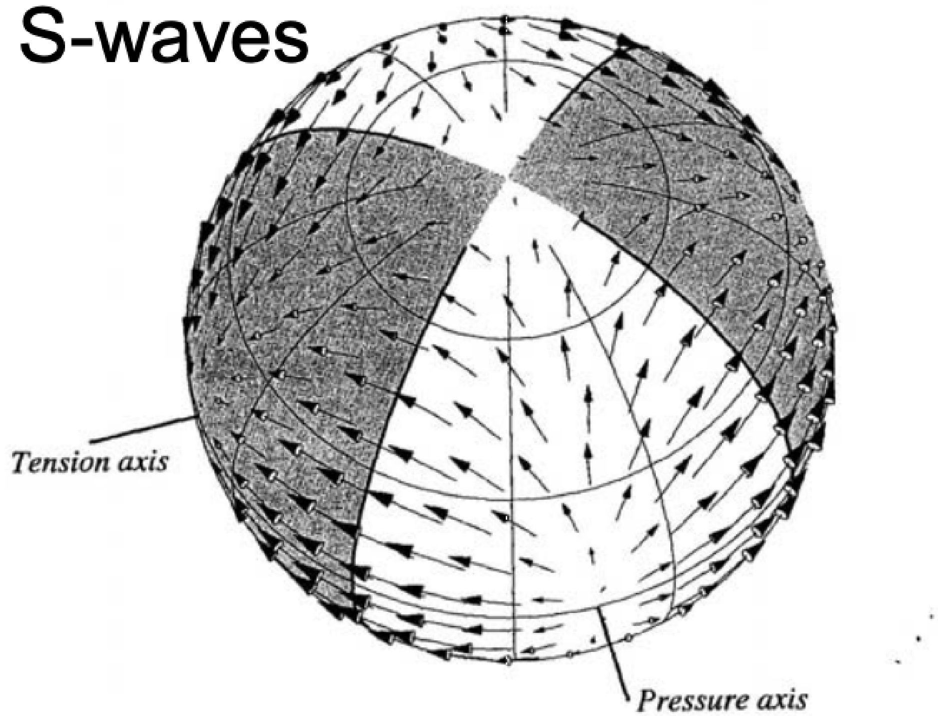
Beach Balls



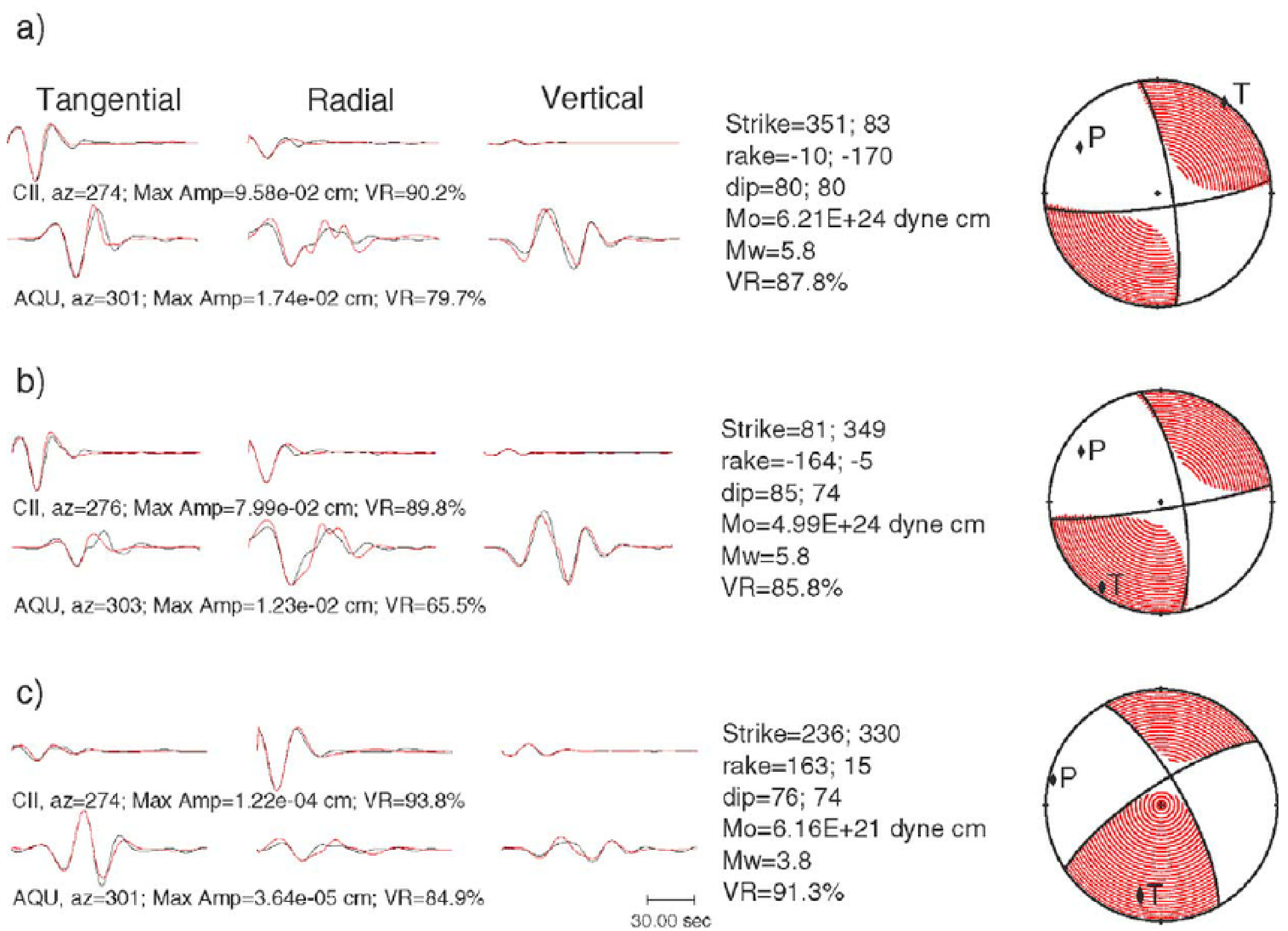
dilatation
compression
compression
dilatation
Hypocenter
Equatorial plane
P-waves
dilatation
compression
compression
dilatation
Seismographic Stations

Relation with Radiation Pattern

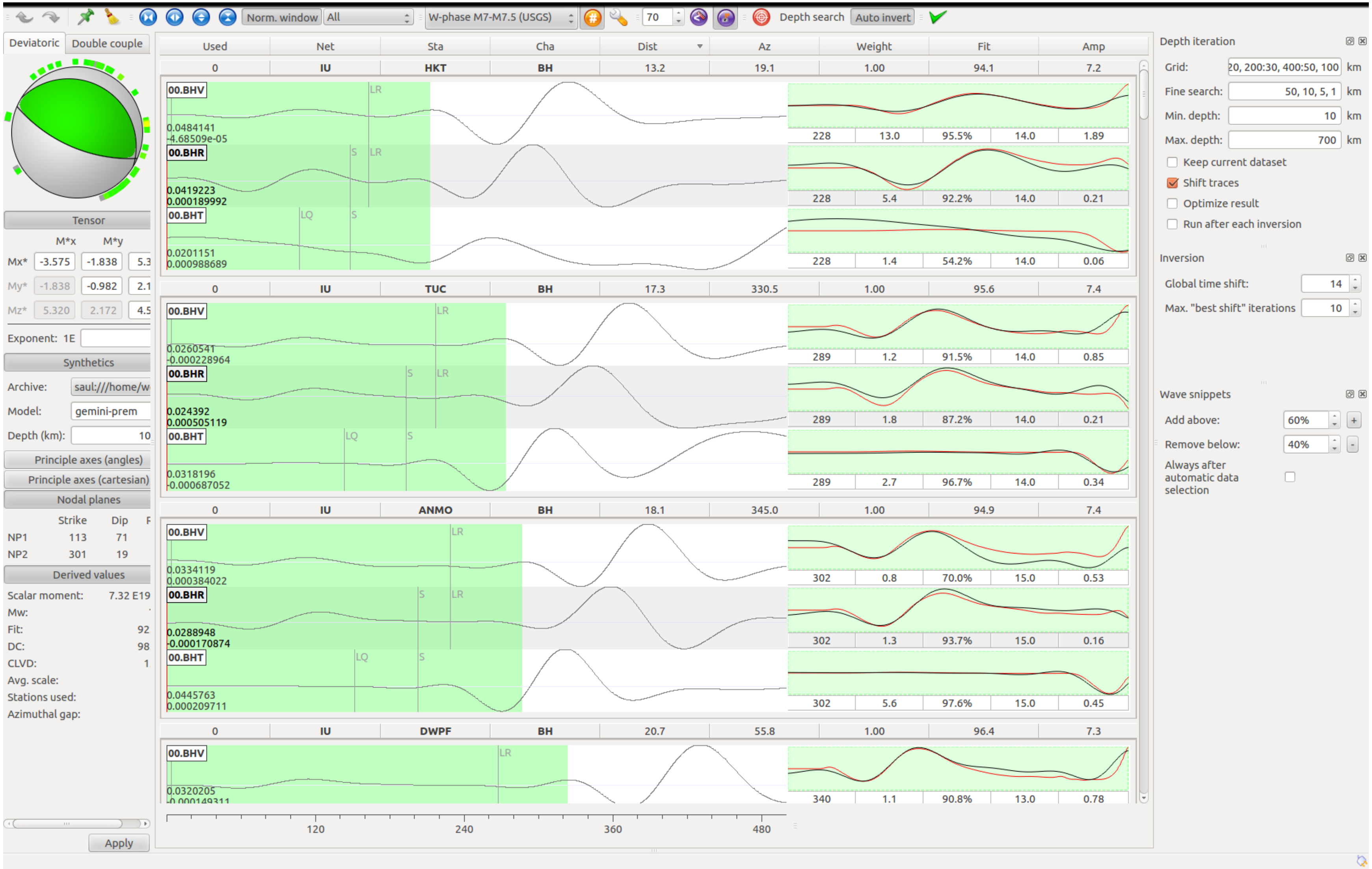




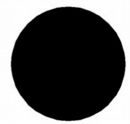
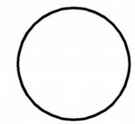
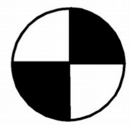
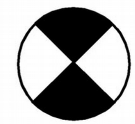
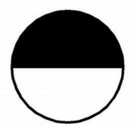
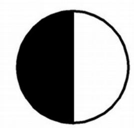
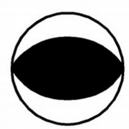
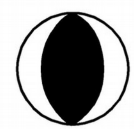
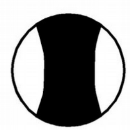
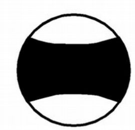
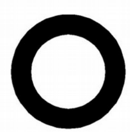
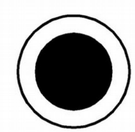
Moment Tensor Inversion



Specialized Software



Relation with Moment Tensor

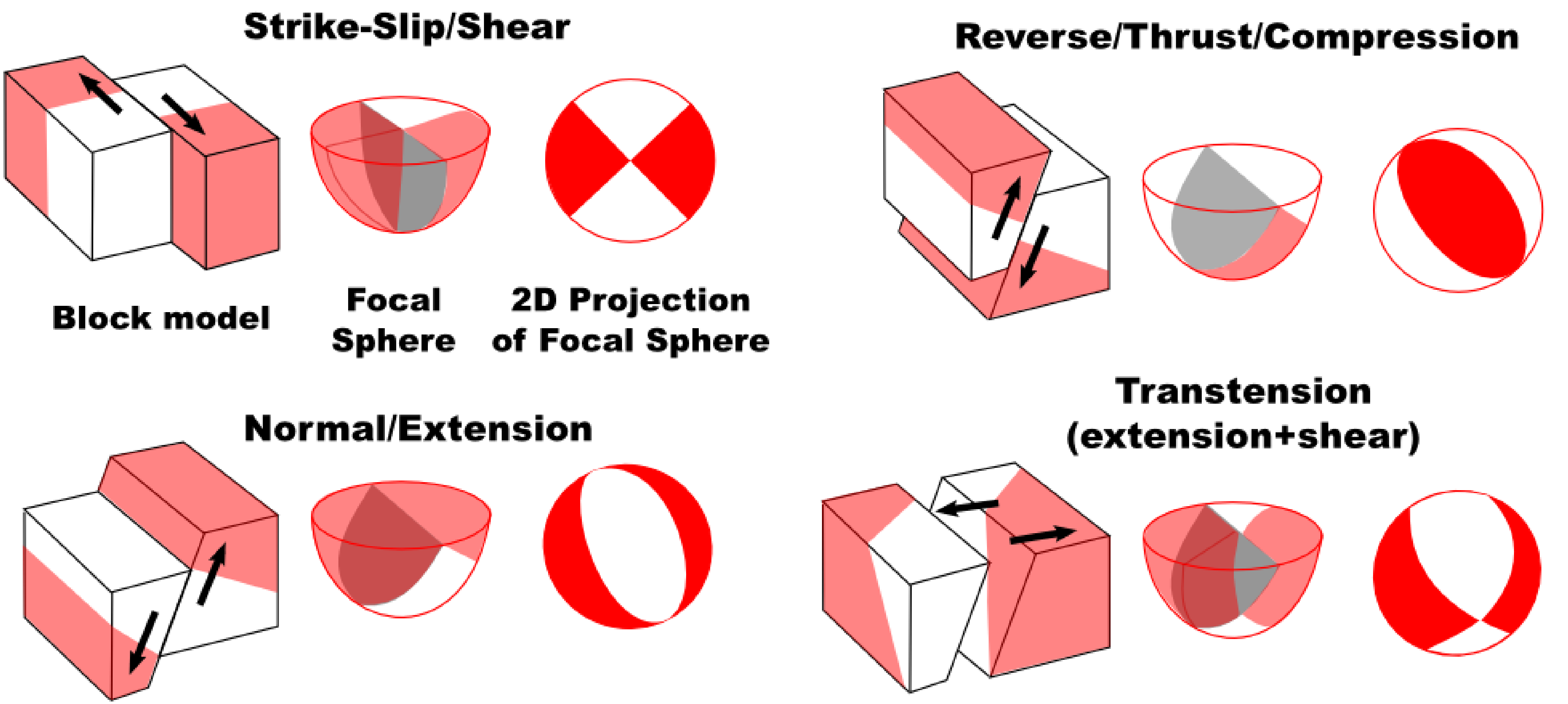
Moment tensor	Beachball	Moment tensor	Beachball
$\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$		$-\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	
$-\frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}$	
$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&-1\\0&0&0\\-1&0&0\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0\\0&0&-1\\0&-1&0\end{pmatrix}$	
$\frac{1}{\sqrt{2}}\begin{pmatrix}-1&0&0\\0&0&0\\0&0&1\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0\\0&-1&0\\0&0&1\end{pmatrix}$	
$\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0\\0&-2&0\\0&0&1\end{pmatrix}$		$\frac{1}{\sqrt{6}}\begin{pmatrix}-2&0&0\\0&1&0\\0&0&1\end{pmatrix}$	
$\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$		$-\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$	

Triple dipole
(e.g. explosions)

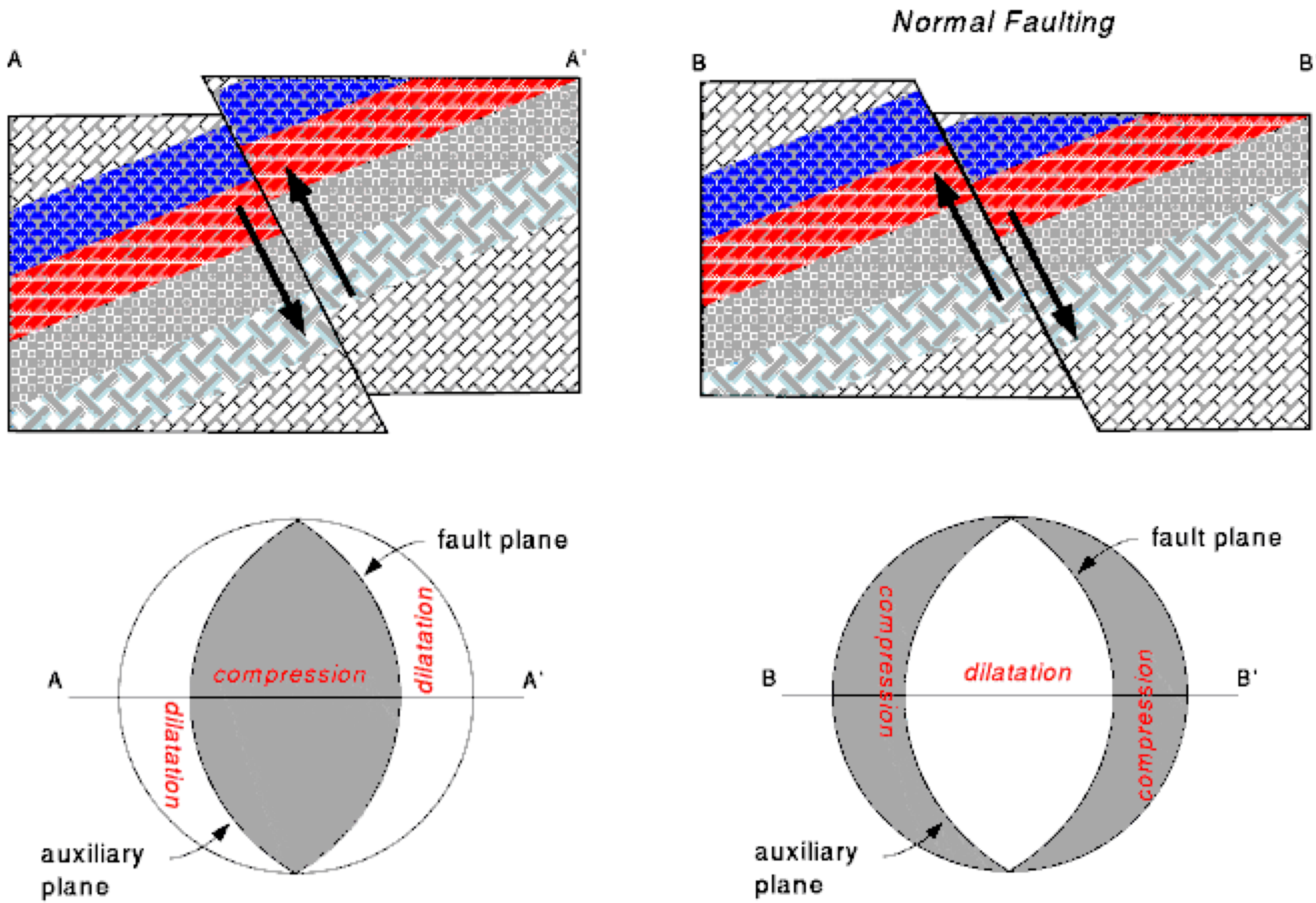
Double couple
(e.g. faults)

Mixed
(e.g. volcanic
eruptions

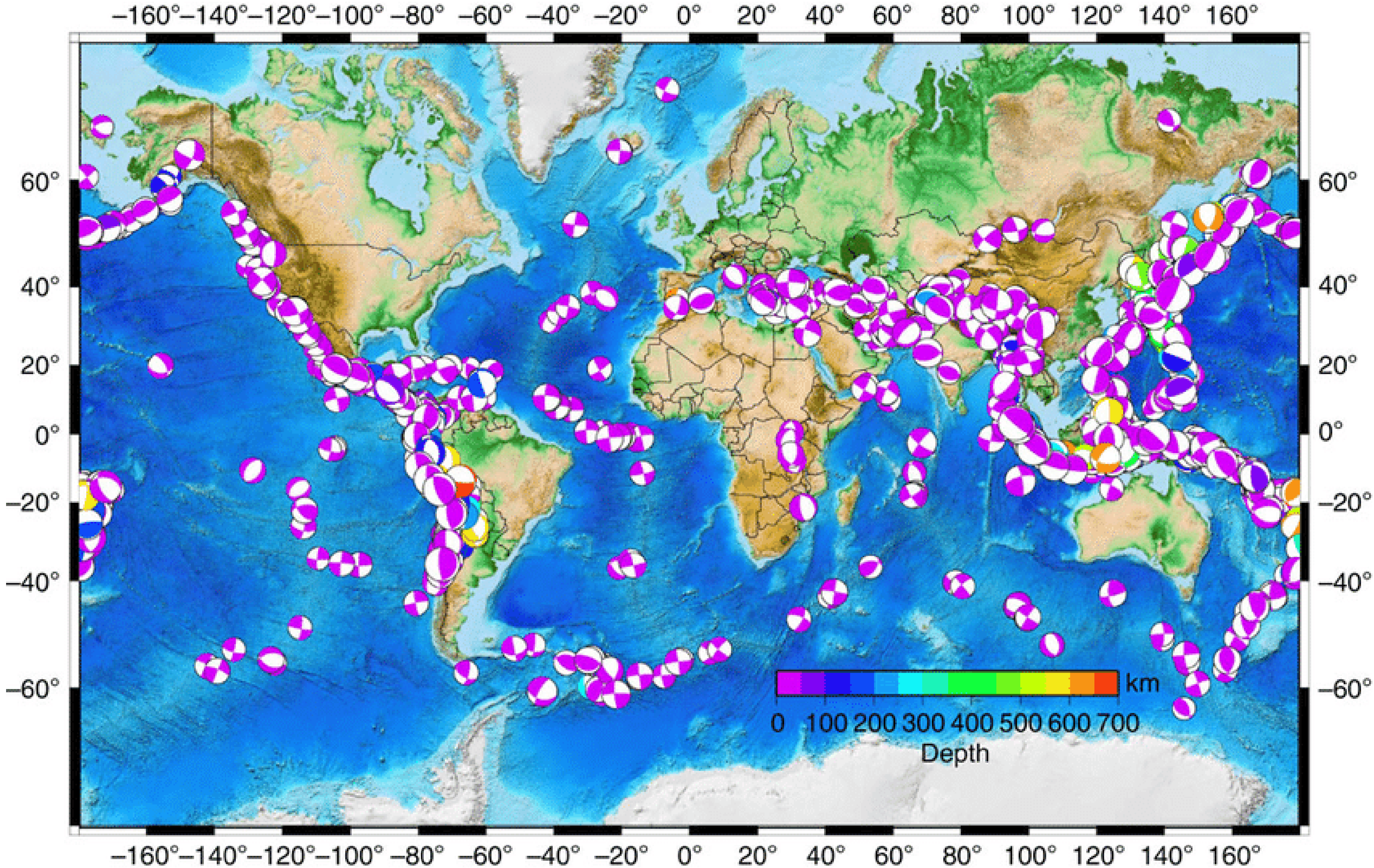
Some Examples



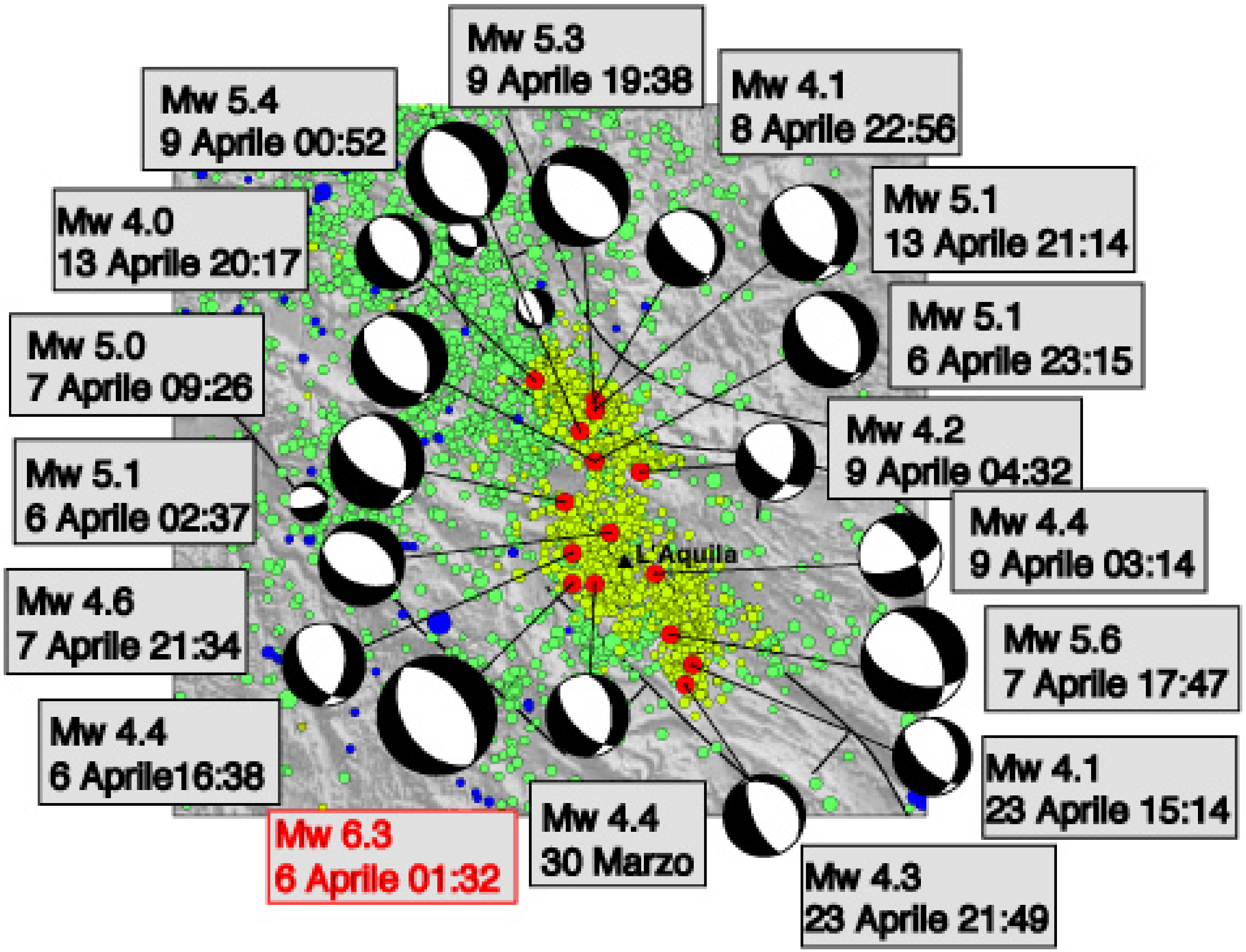
Some Examples



Focal Mechanisms in the World

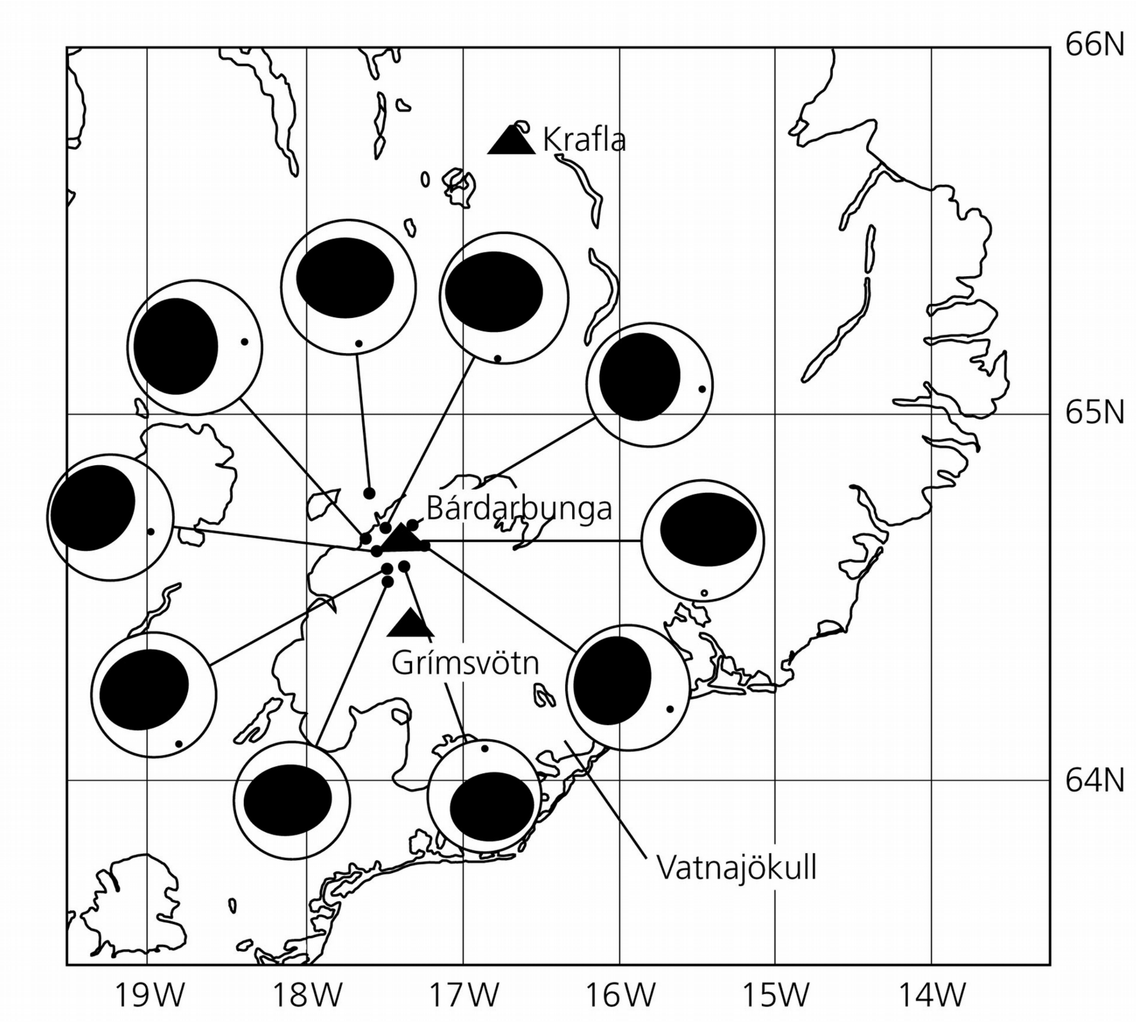


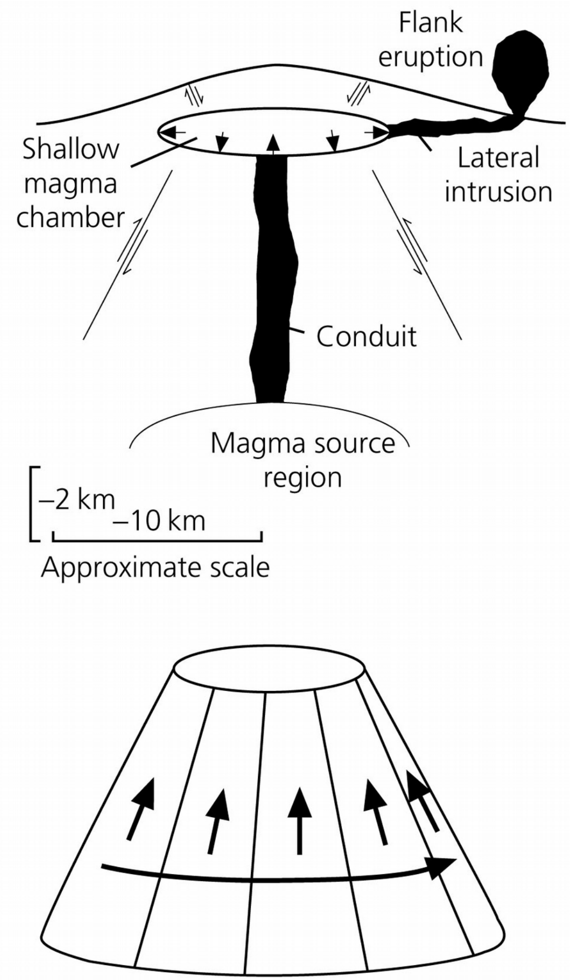
The Example of l'Aquila Sequence



Volcanic Earthquakes

Few earthquakes types are not generated by faults (no double couple mechanism) and have typical signature





Far-Field Solution Recap

Isolating the S-wave far-field term of the double couple solution we get:

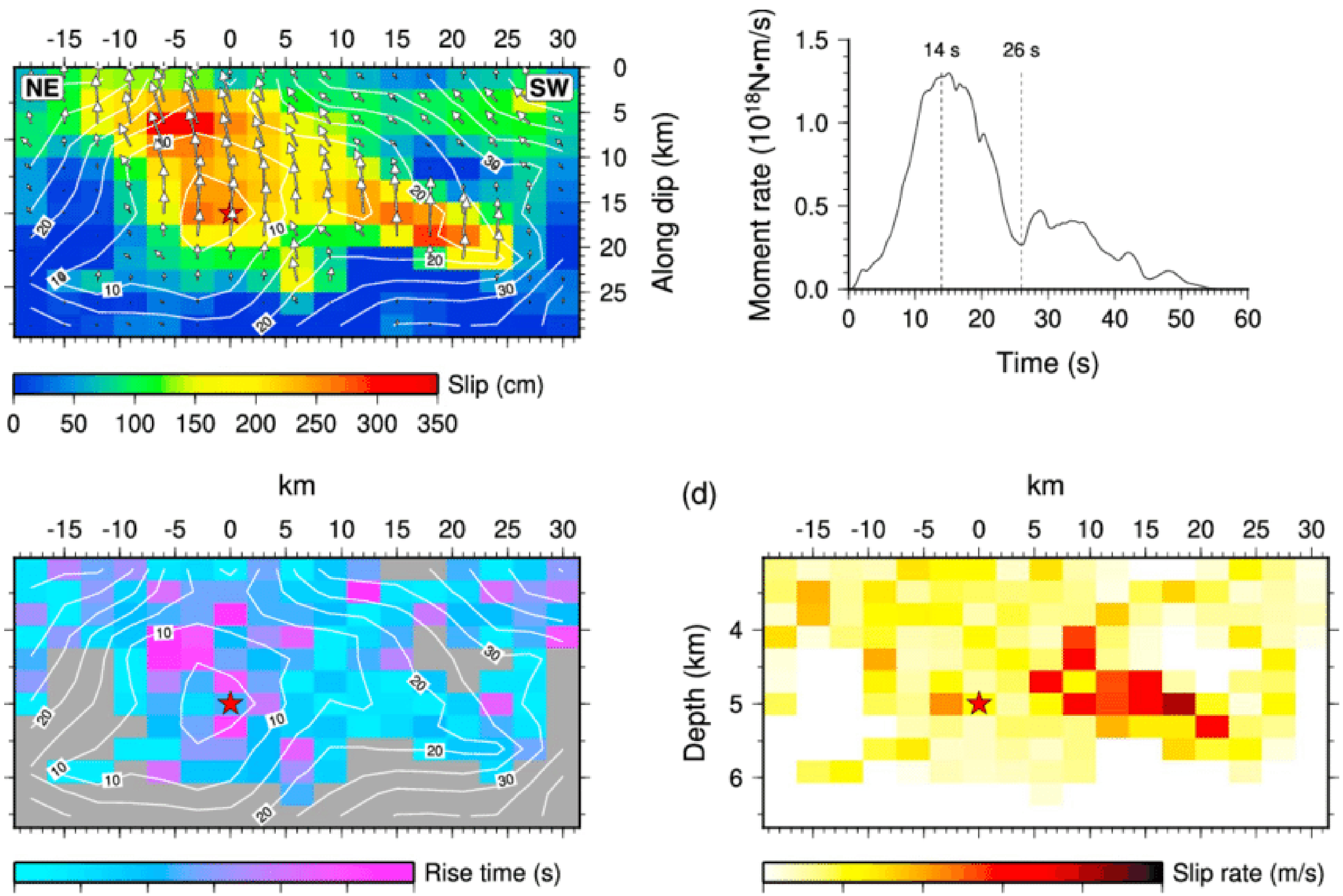
$$u_i^S(x,t)=\frac{1}{4\pi\rho\beta^3}R_{ijk}^{FS}\frac{1}{r}\frac{\delta M_{jk}}{\delta t}\left(t-\frac{r}{\beta}\right)$$

Semplifying, the far-field displacement is proportional to the derivative of the seismic moment, and thus to the derivative of the slip time function (slip velocity):

$$u^S \propto C\dot{M}_0 \propto C\mu A\dot{D}$$

Finite Faults

So far we have considered the case of point sources, however a real fault has a finite extension. In this case, the slip has duration which depends on the propagation of the rupture along the fault.



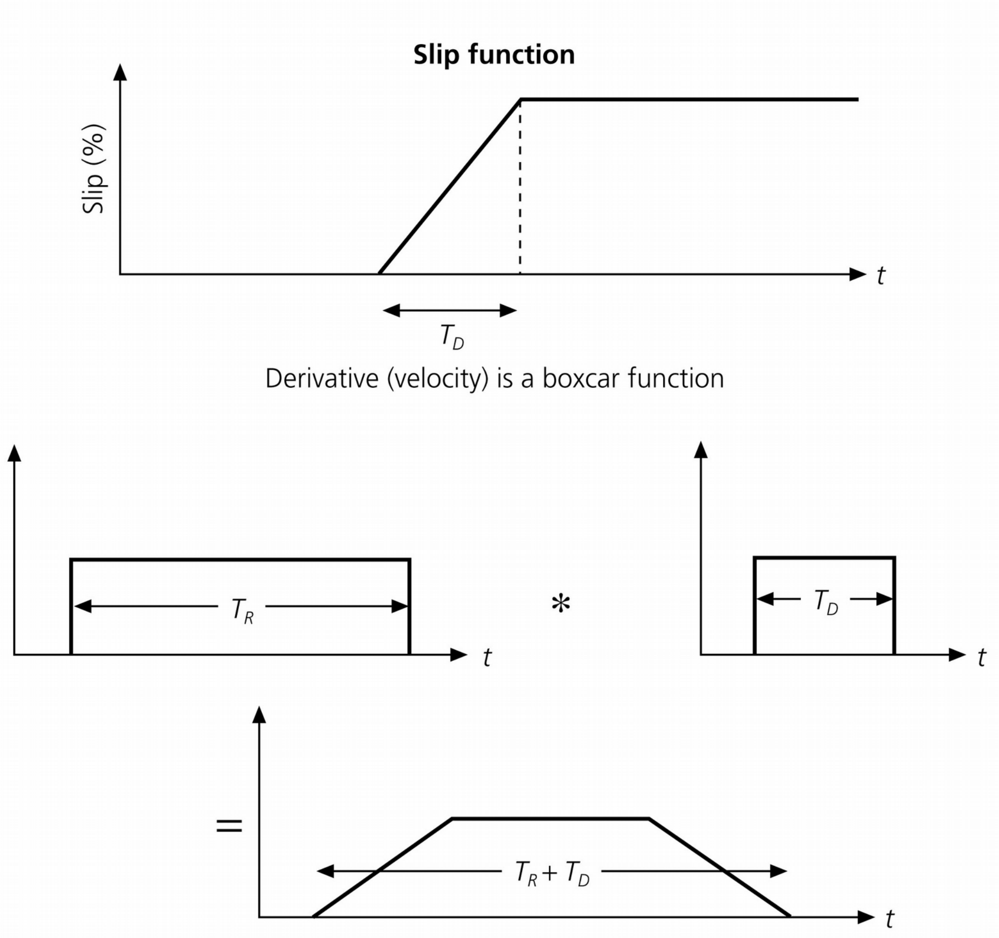
Source Time Function

On fault of finite extension, slip at a given point does not occur instantaneously. The slip history can be modeled as ramp function of T_R, the rise time.

The source time function depends on the time derivative of the slip history, which is a boxcar.

When convolved with the boxcar time function due to rupture propagation (with duration T_D), the resulting source time function is trapezoidal.

The width of the source function is then equal to the sum of the **rupture duration (T_D) and rise (T_R) time**.



(Far-Field) Source Spectrum

The displacement <u>on a finite fault</u> can be written as the convolution of two boxcar functions, scaled by seismic moment:

$$u(t) = M_0 B(t, T_D) * B(t, T_R)$$

In frequency domain, the transform of a boxcar of high 1/T and length T is a sinc function. Thus, the spectral amplitude of the source signal is the product of the seismic moment of two sinc terms:

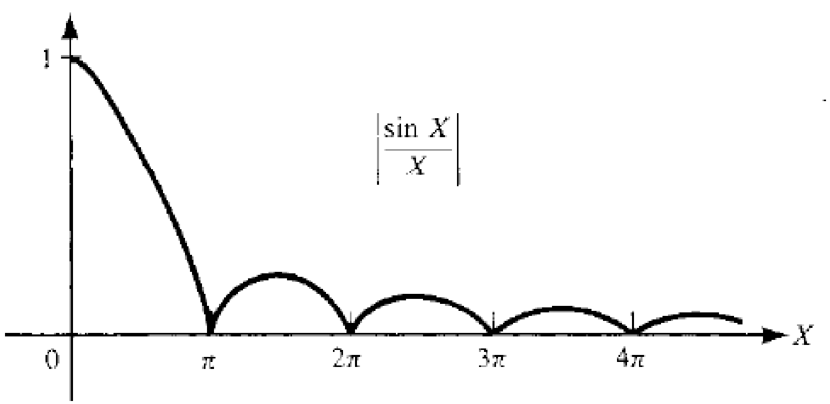
$$|A(\omega)| = M_0 \left| \frac{\sin(\omega T_R/2)}{\omega T_R/2} \right| \left| \frac{\sin(\omega T_D/2)}{\omega T_D/2} \right|$$

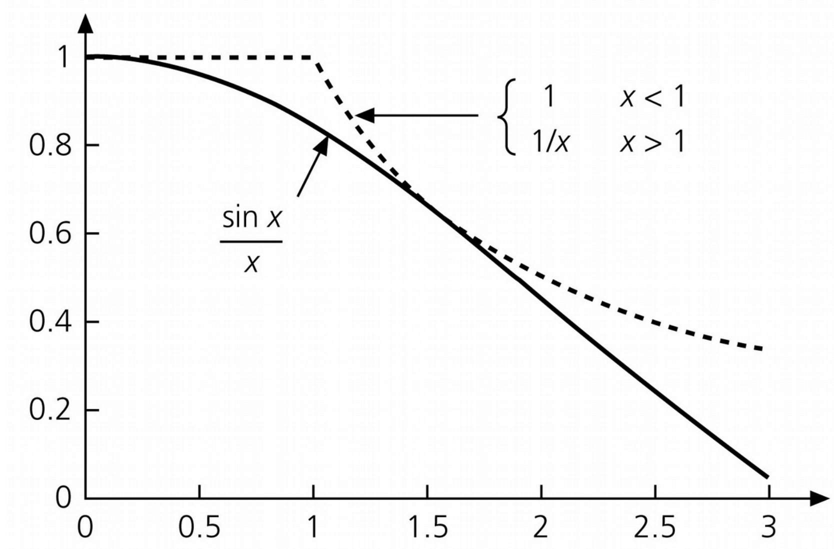
Where T_R and T_D are the **rupture and rise time**, respectively

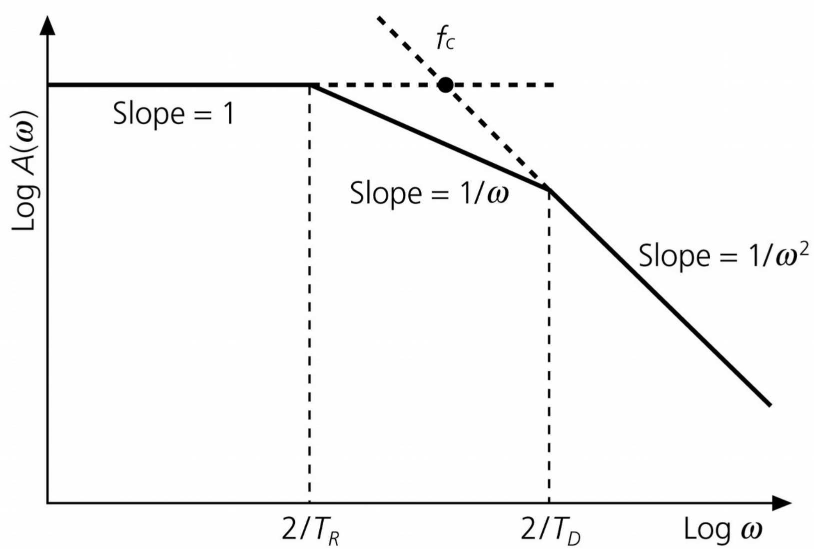
Source Spectrum

It is often useful to approximate sinc(x) as a piecewise line function, e.g. 1 for x<1 and as 1/x for x>1.

The hinge point of the two parts is the corner frequency, which depends on the fault size.

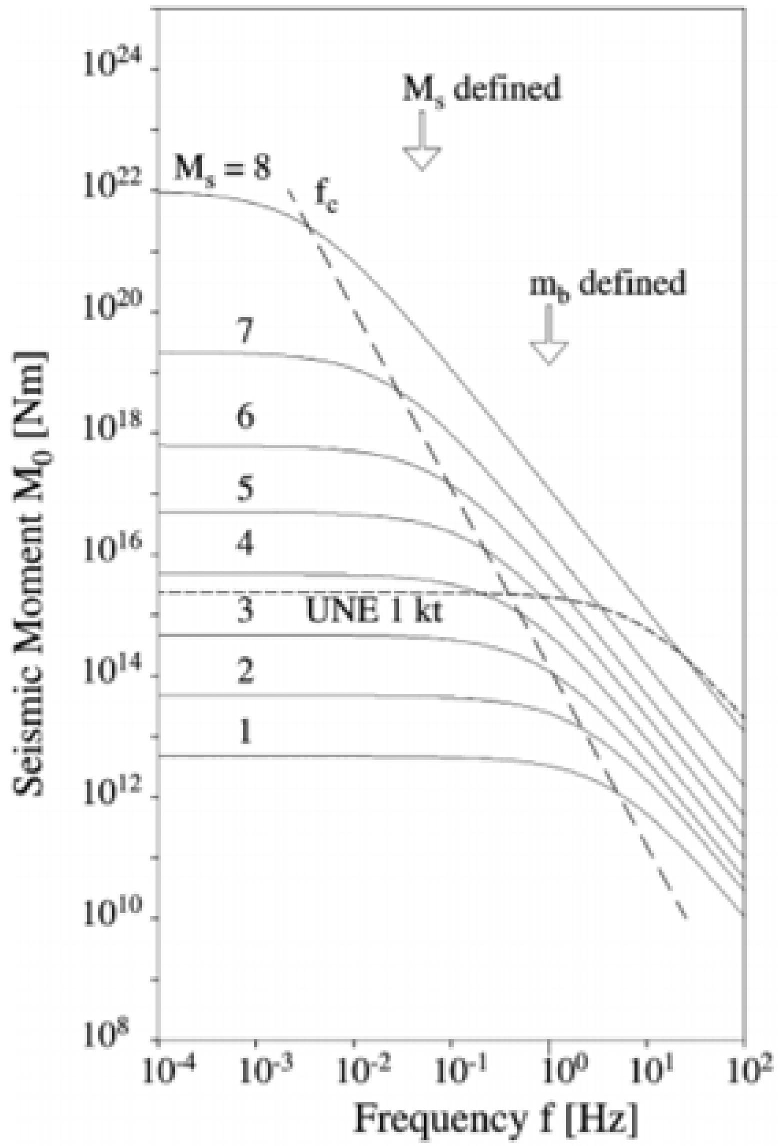






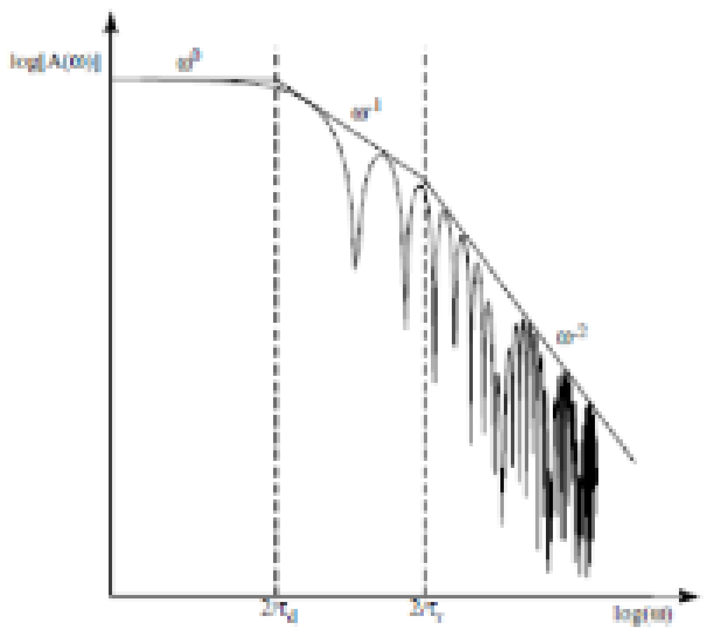
Source Spectrum

As rupture lengths increase, the seismic moments, rise times and rupture durations increase. Thus, the corner frequencies move to the left, i.e. to lower frequencies.

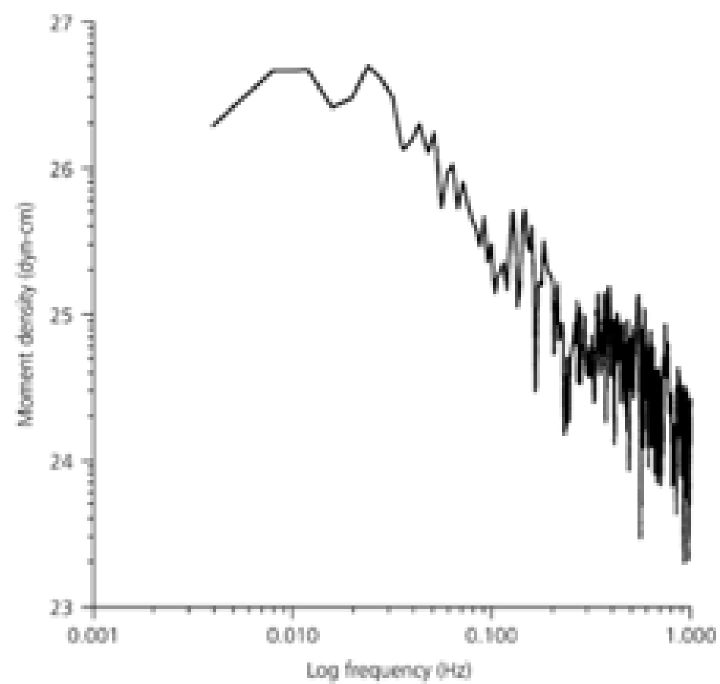


Source Spectrum

While in principle, by studying the spectra of real earthquakes, we can recover M_0, T_R and T_D, in practice things are more complicated than that.



Theoretic spectrum

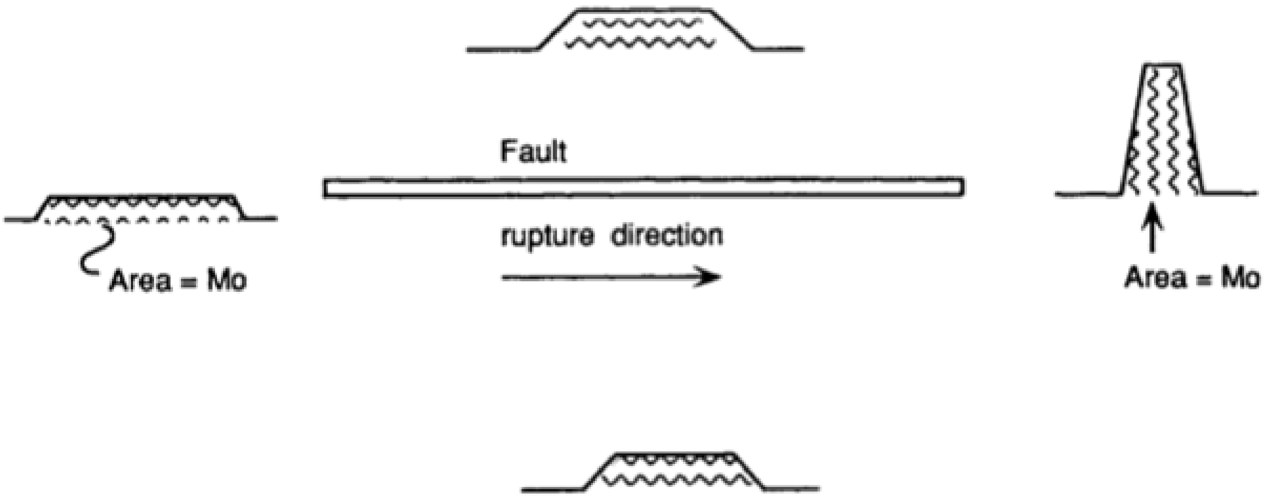


Real spectrum

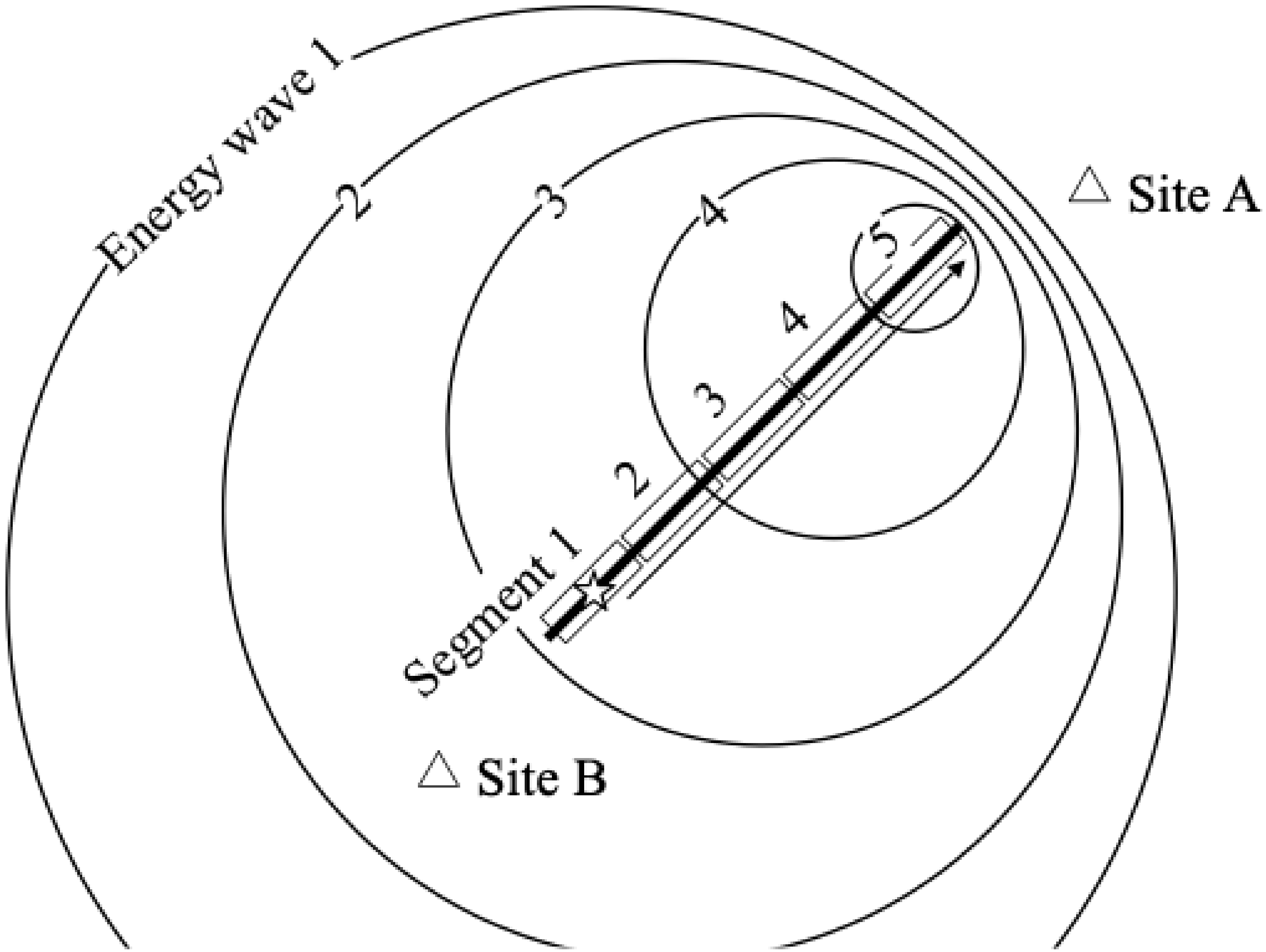
Source Directivity

Directivity is a consequence of a moving source along a fault plane. Waves from far-end of fault will pile up with waves arriving from near-end of fault, if you are forward of the rupture.

This causes increased amplitudes in direction of rupture propagation, and decreased duration. Frequency content is higher, as the majority of the seismic energy is delivered in a large velocity pulse.

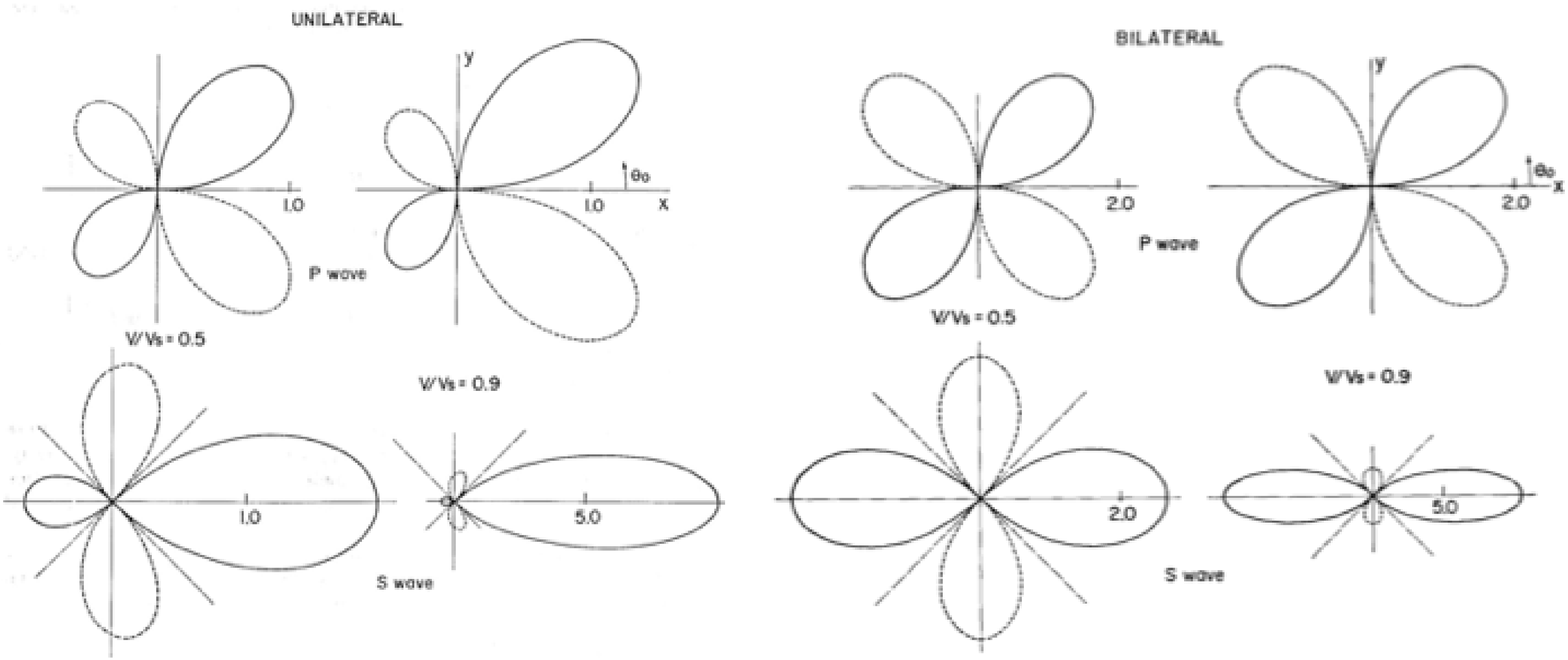


Source Directivity



Directivity Effect on Radiation

Directivity is useful in distinguishing earthquake fault plane from its auxiliary plane because it destroys the symmetry of the radiation pattern.



Directivity Example

Strong motion recordings of the Landers 1992, M=7.3 earthquake at Lucerne station.